Friendly Corecursion in Isabelle/HOL

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Overview

Part 1: What Is

Part 2: What We Do

Part 1: What Is

Running Example: Streams of Numbers

$\mathsf{Str} = \mathbb{N}^{\mathbb{N}}$

Use x, y, z for numbers and xs, ys, zs for streams (of numbers)

Each stream xs has the form $xs_0, xs_1, xs_2, \ldots$

Constructor $\# : \mathbb{N} \times \mathsf{Str} \to \mathsf{Str}$

Selectors $\mathsf{hd} : \mathsf{Str} \to \mathbb{N}$, $\mathsf{tl} : \mathsf{Str} \to \mathsf{Str}$

Destructor $\$ = \langle \mathsf{hd}, \mathsf{tl} \rangle : \mathsf{Str} \to \mathbb{N} \times \mathsf{Str}$

$\#$ and $\$$ are mutually inverse bijections.

$$\oplus : \mathsf{Str}^2 \to \mathsf{Str}$$

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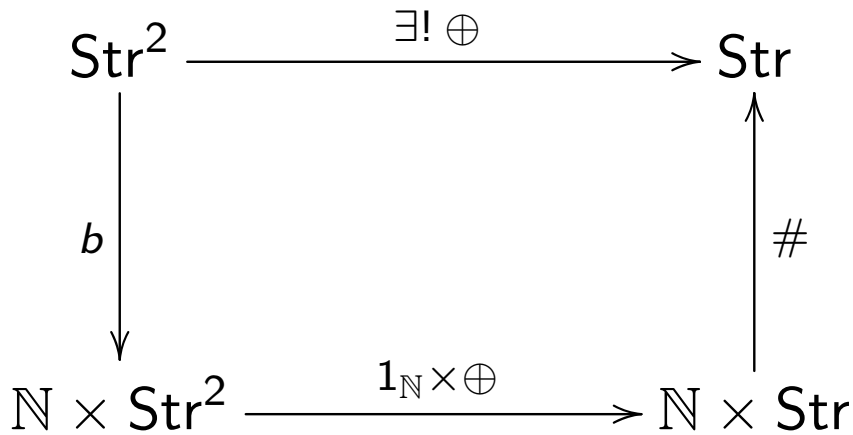

where $b = \lambda(xs, ys).\ (\mathsf{hd}(xs) + \mathsf{hd}(ys),\ (\mathsf{tl}(xs), \mathsf{tl}(ys)))$
is $\oplus$'s "blueprint"

Primitive Corecursion

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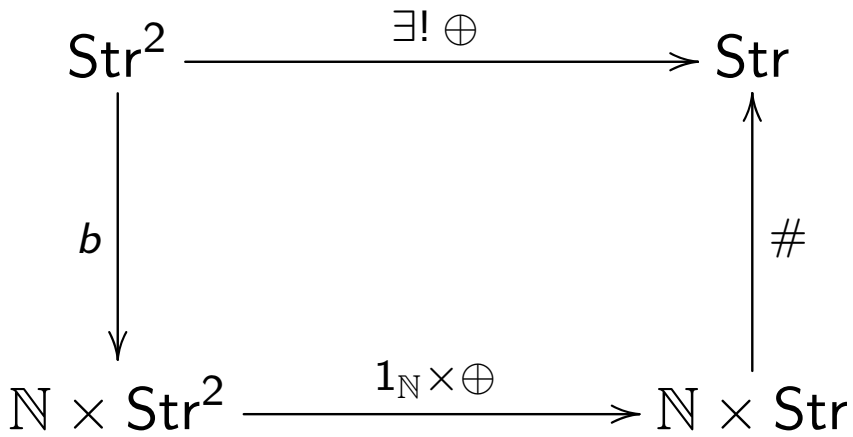

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$$\odot : \mathsf{Str}^2 \to \mathsf{Str}$$

$$\begin{array}{lcl} xs \odot ys & = & (\mathsf{hd}(xs) \cdot \mathsf{hd}(ys)) \,\# \\ & & (xs \odot \mathsf{tl}(ys)) \oplus (\mathsf{tl}(xs) \odot ys) \end{array}$$

Corecursion Up To $\oplus$

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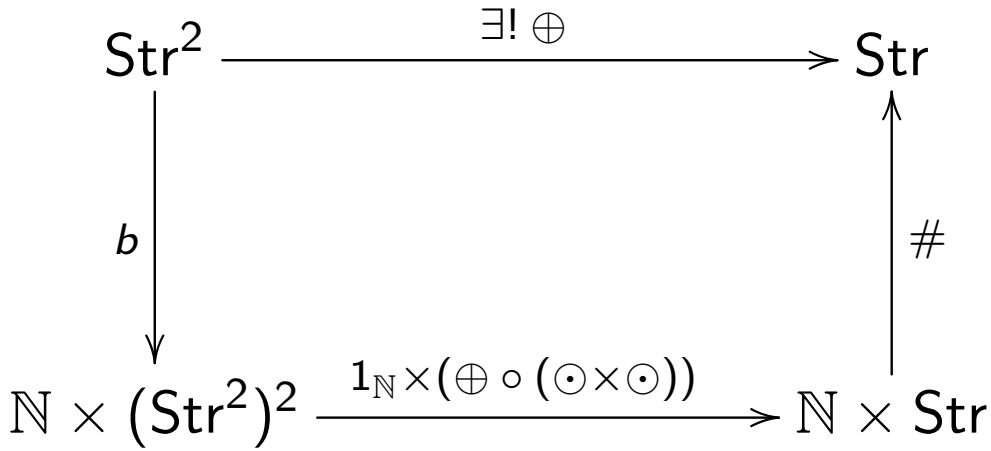

$$\begin{aligned} \text{where } b &= \lambda(xs, ys).\ \ (\mathsf{hd}(xs) \cdot \mathsf{hd}(ys), \\ &\qquad\qquad ((xs, \mathsf{tl}(ys)), (\mathsf{tl}(xs), ys))) \end{aligned}$$

$$f : \mathsf{Str}^3 \to \mathsf{Str}$$

$$f(xs, ys, zs) = \text{let } us = \mathsf{tl}(\mathsf{tl}(\mathsf{hd}(xs)\#\mathsf{zip}(zs, \mathsf{tl}(\mathsf{tl}(xs))))) \text{ in}$$
$$(\mathsf{hd}(us) \cdot \mathsf{hd}(\mathsf{tl}(ys))) \#$$
$$((f(xs, zs, us) \odot us) \oplus (f(xs, xs, ys) \oplus f(us, ys, zs)))$$

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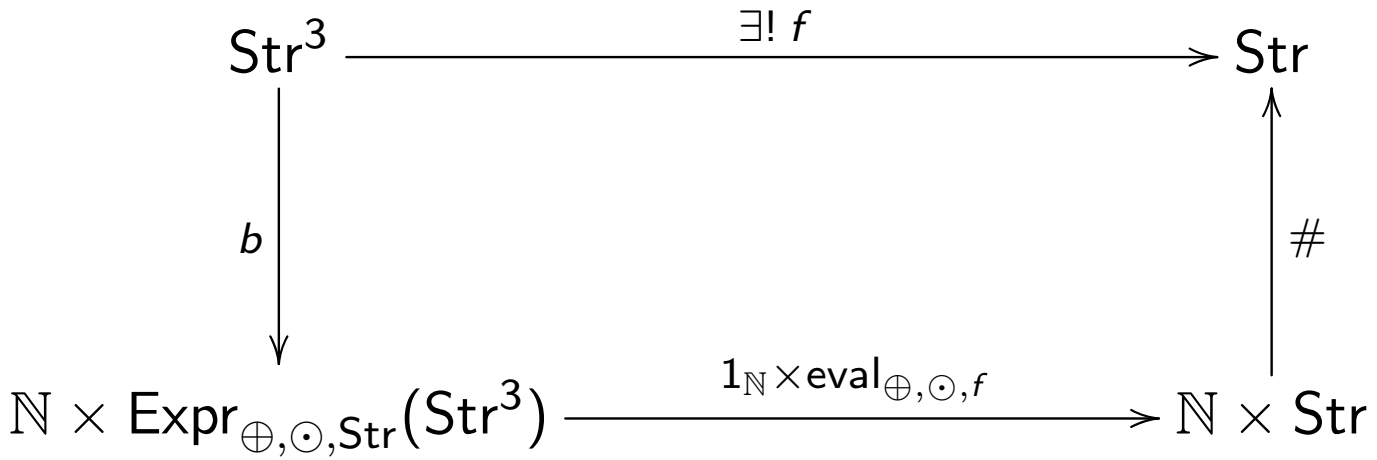

$\mathrm{Expr}_{\oplus,\odot,\mathrm{Str}}(X)$ formal expressions with $\oplus$, $\odot$, consts Str, vars X

$\mathrm{eval}_{\oplus,\odot,f} : \mathrm{Expr}_{\oplus,\odot,\mathrm{Str}}(\mathrm{Str}^3) \to \mathrm{Str}$ evaluator

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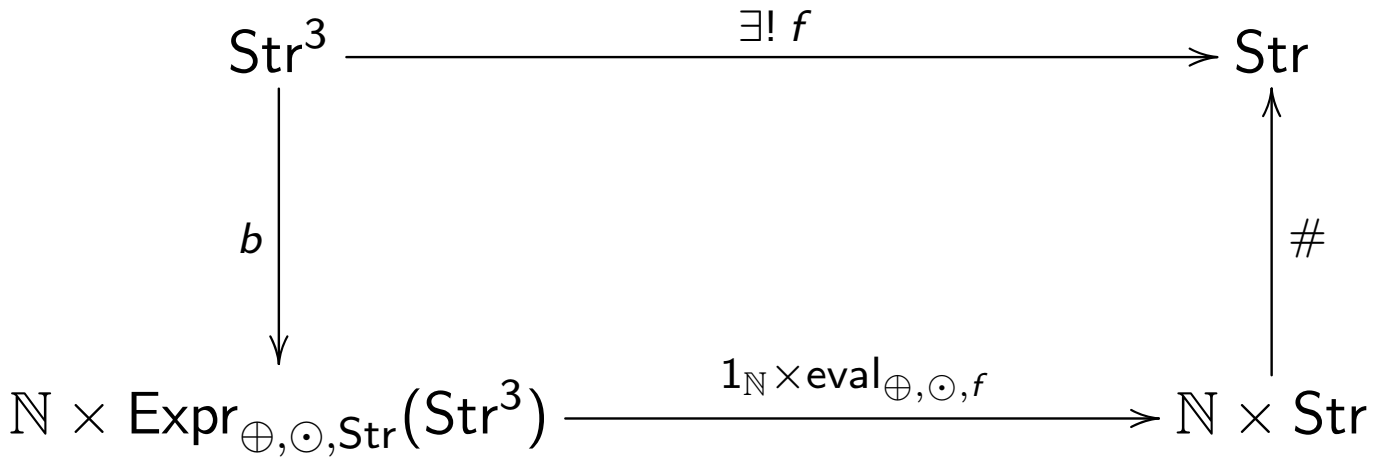

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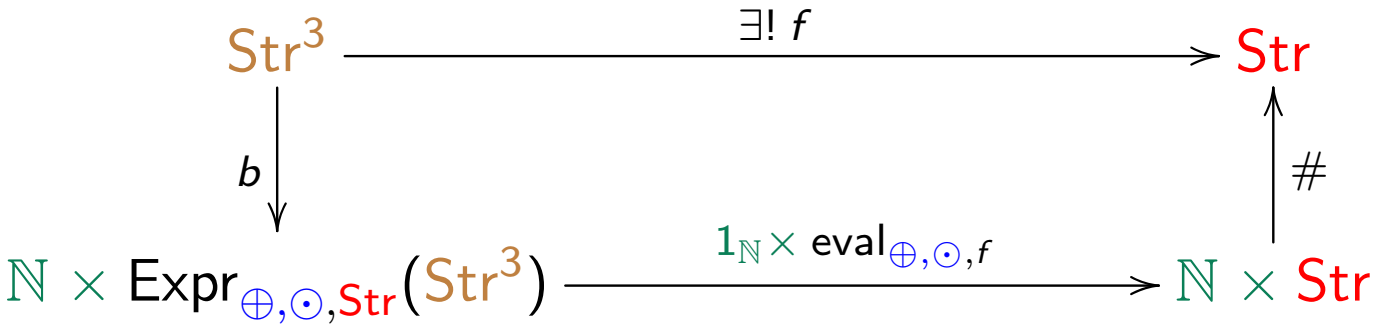
$$(\mathrm{Var}(xs, zs, us) \boxed{\odot} \mathrm{Ct}(us)) \boxed{\oplus} (\mathrm{Var}(xs, xs, ys) \boxed{\oplus} \mathrm{Var}(us, ys, zs))$$

Corecursion Up To $\{\oplus, \odot\}^*$: Generalization

Given up-to-friendly operators $\oplus : \mathrm{Str}^2 \to \mathrm{Str}$, $\odot : \mathrm{Str}^2 \to \mathrm{Str}$
... can define $f : \mathrm{Str}^3 \to \mathrm{Str}$ by choosing a blueprint b:



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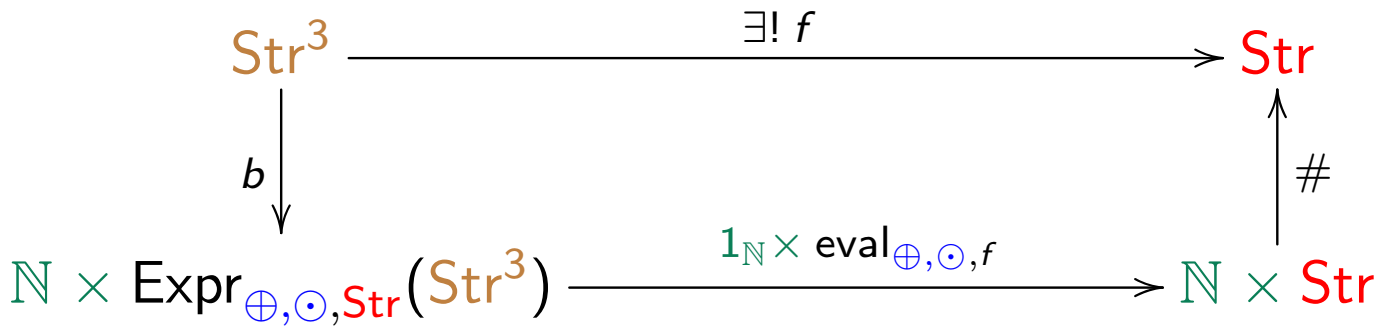

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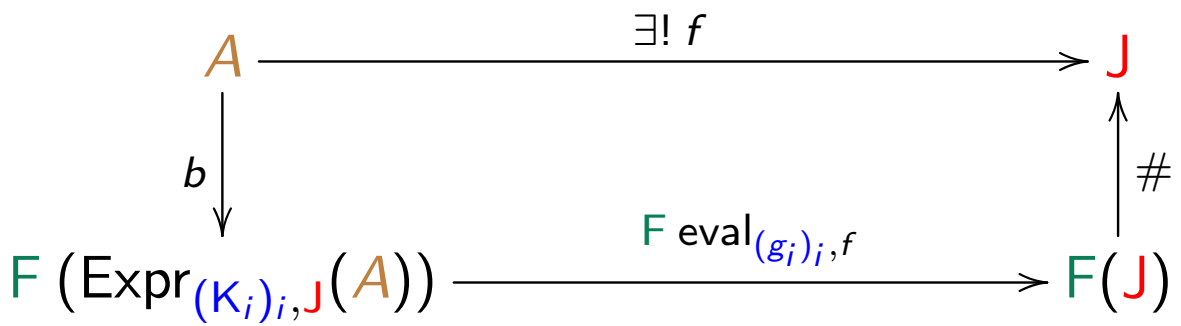

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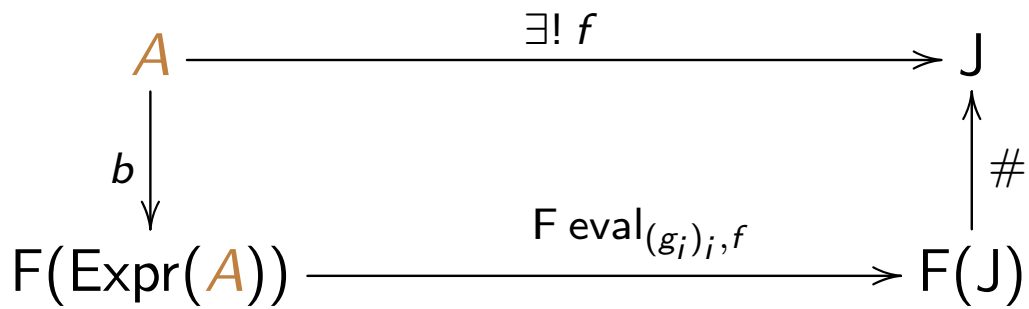

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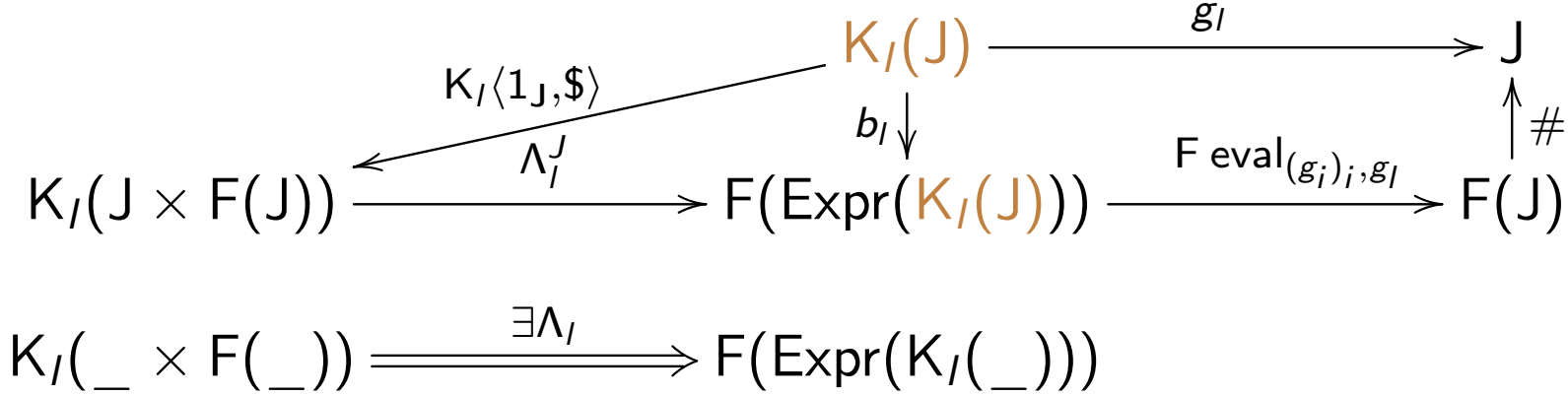
Up-To-Friendly Operators: Consume ≤ 1, Produce ≥ 1



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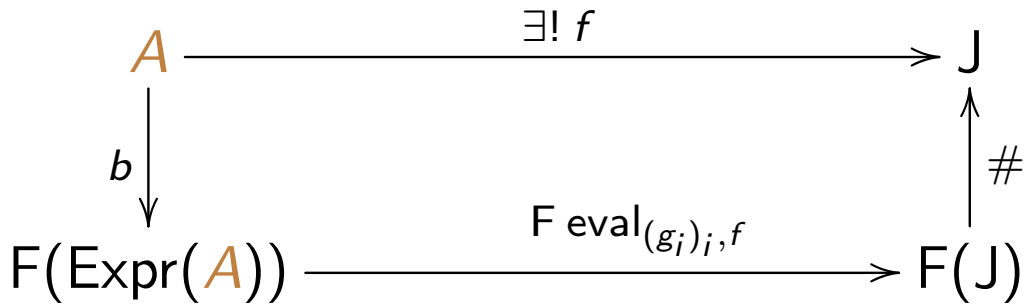

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Hence could replace F Expr with Expr

F is important though, since it acts as a guard. But the guard needs not be at the top: Define GExpr = Expr F Expr

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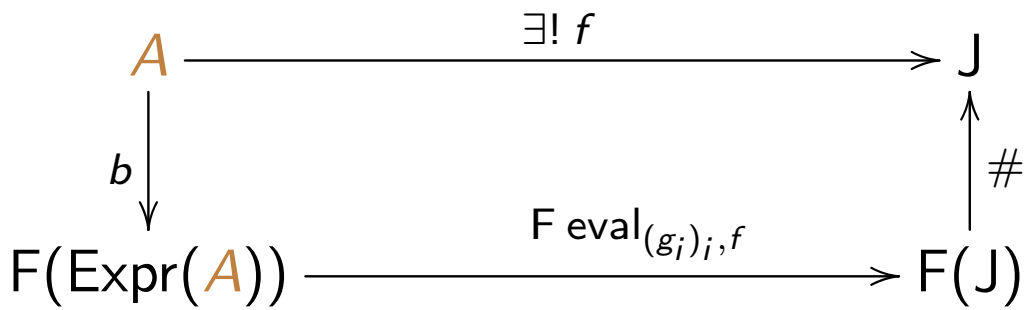

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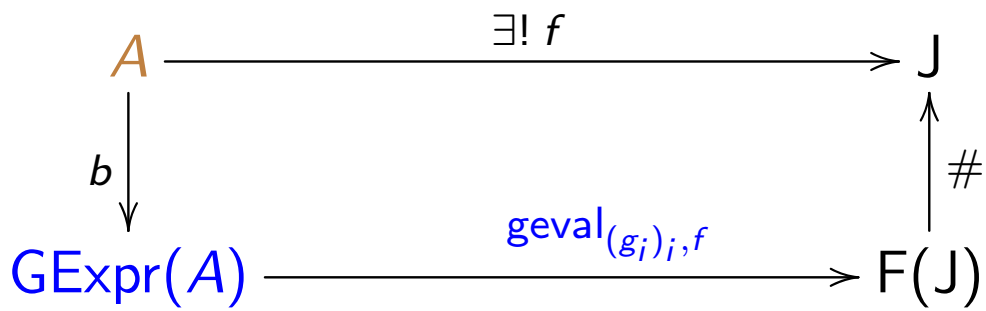

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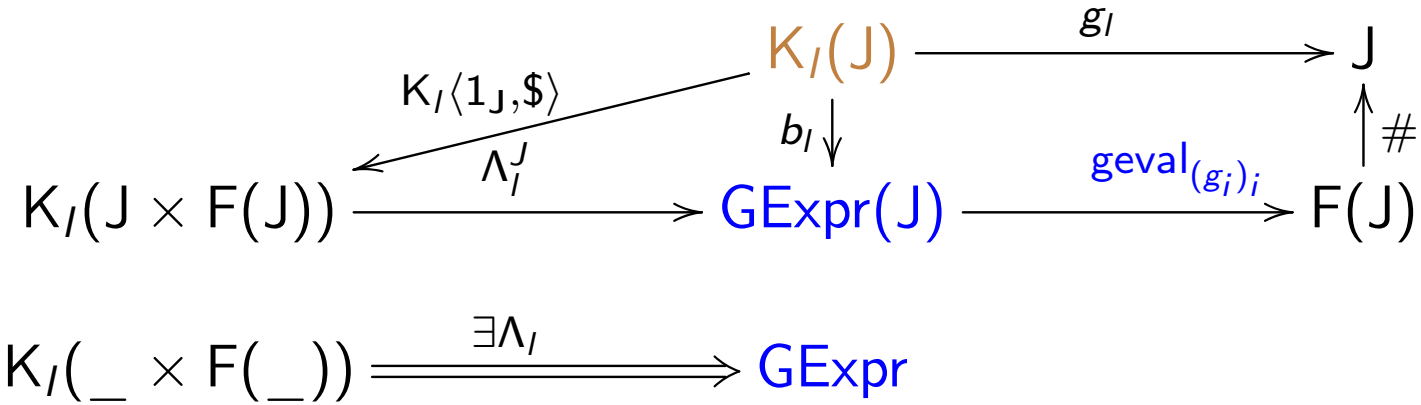

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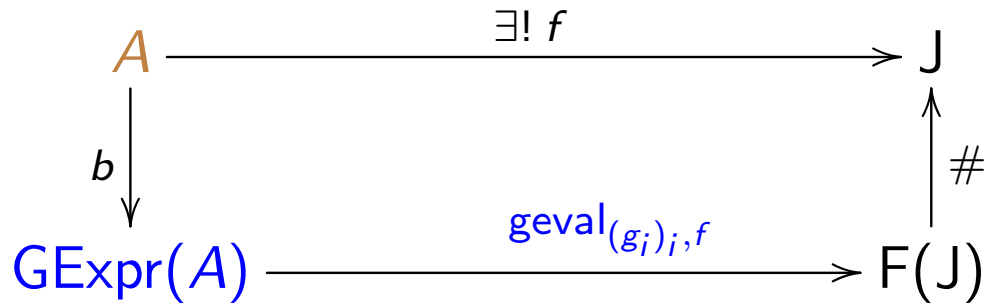

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Part 2: What We Do

What We Do in Isabelle/HOL

For each defined codatatype J, remember its defining functor F (s.t. J is its final coalgebra).

codatatype stream = Cons (hd : nat) (tl : stream)
Remember functor nat $\times$ α

Store friendly operators and their distributive laws – forming the current "corecursion state". Start with the constructor(s).

Register Cons as friendly
Prove up-to-Cons corecursion theorem

Functions targeting J can be defined by the corecursion theorem.

corec $\oplus$: stream $\to$ stream $\to$ stream
where *xs* $\oplus$ *ys* = Cons (hd *xs* + hd *ys*) (tl *xs* $\oplus$ tl *ys*)
Synthesize suitable blueprint and use it to define $\oplus$

Some functions K(J) $\to$ J can be registered as friendly

friend_of_corec $\oplus$
Synthesize distributive law and prove naturality and friendliness
Prove stronger corecursion theorem, factoring in the new friend

What We Do in Isabelle/HOL: Reasoning

Isabelle keeps track of Bounded Natural Functors (BNFs) – k-accessible functors that preserve weak pullbacks.

Hence are relators: Every $R \subseteq A \times B$ lifted to $\mathsf{Frel}(R) \subseteq \mathsf{F}\ A \times \mathsf{F}\ B$.

Full abstraction: If $\mathsf{J} \xrightarrow{\$} \mathsf{F}(\mathsf{J})$ is F's final coalgebra, then equality on J is the largest F-bisimulation.

$$\Downarrow$$

Proof principle: Given $R \subseteq J \times J$,

if $\forall a, b.\ (a, b) \in R \longrightarrow (\$\ a, \$\ b) \in \mathsf{Frel}(R)$

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if $\forall a, b.\ (a, b) \in R \longrightarrow (\$\ a, \$\ b) \in \mathsf{Frel}(\mathsf{Cong}_{(g_i)_i}(R))$

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$\mathsf{Cong}_{(g_i)_i}(X)$ defined (inductively) as the smallest equivalenvce that includes X and is friend-compatible:

$(k, k') \in \mathsf{Krel}_I(\mathsf{Cong}_{(g_i)_i}(X)) \longrightarrow (g_I(k), g_I(k')) \in \mathsf{Cong}_{(g_i)_i}(X)$

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Coinduction principle updated together with the corecursion state.

Things We Don't Do in Isabelle/HOL

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Question by user on a mailing list: "I have a coinductive predicate P and I want to show that "P s" holds, where "s" is a corecursively-defined stream ... How do I do something like this?"

My answer: [Category-theory based suggestion, comprising three steps.]

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My answer: [Category-theory based suggestion, comprising three steps.]

His follow-up, also making three points: "I have decided not to look into this any further at the moment, seeing as

1. it gives me a headache,
2. I have working proofs for now and
3. I'm quite eager to continue my formalisation elsewhere."

Things We Don't Do in Isabelle/HOL

While benefiting from category theory, users are not exposed to it:

Blueprints and distributive laws are inferred automatically and disappear after doing their jobs.

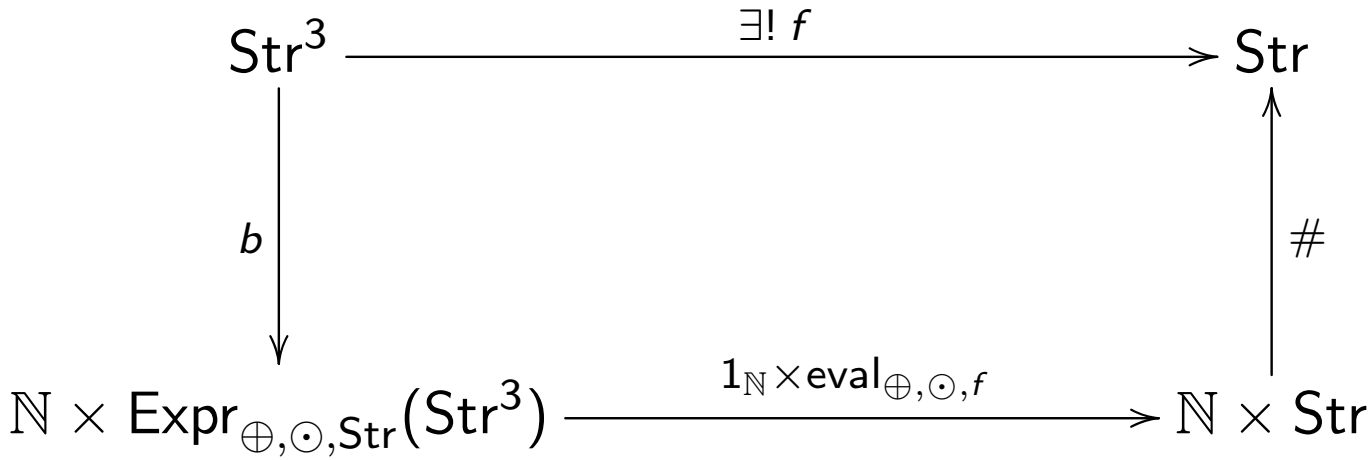
Reasoning infrastructure does not need them either: It works directly with the concrete operators.

Example

User writes equation, counting on guarded computation intuition:

$$
\begin{aligned}
&f(xs, ys, zs) \;=\\
&\quad (\mathsf{hd}(xs) \cdot \mathsf{hd}(\mathsf{tl}(ys)))\,\#\\
&\quad ((f(xs, zs, xs) \odot xs) \oplus (f(xs, xs, ys) \oplus f(zs, ys, zs)))
\end{aligned}
$$

In response, system silently does the following work:



. . . and comes back accepting that equation as a definition.

Some Literature

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Bartels 2003. Generalised Coinduction

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Abel, 2014. Programming and Reasoning with Infinite Structures Using Copatterns and Sized Types

In Case People Are Not Too Eager To Go To Lunch...

$$\mathsf{f} : \mathbb{N} \to \mathsf{Str}$$

$$\mathsf{f}\,(n) = \begin{cases} \mathsf{zeros} & \text{if } n \leq 1 \\ (n \,\#\, \mathsf{f}\,(n\,\mathsf{DIV}\,2)) \odot (n \,\#\, \mathsf{f}\,(3 * n + 1)) & \text{if } n > 1 \end{cases}$$

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- Friendly context around the calls
- All calls are guarded

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Standard blueprint synthesis fails:

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Not all calls are guarded

We don't give up: Try to define a recursive blueprint. Is $b : \mathbb{N} \to \mathsf{GExpr}(\mathbb{N})$ is correctly defined?

$$b\,(n) = \begin{cases} \mathsf{Ct}\,(\mathsf{zeros}) & \text{if } n \leq 1 \\ b\,(n\,\mathsf{DIV}\,2) \boxed{\odot} (n \boxed{\#} \mathsf{Var}\,(3 * n + 1)) & \text{if } n > 1 \end{cases}$$

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It is! By terminating recursion on n. And the function defined from blueprint b turns out to satisfy the desired equation for f.

Eventual Up-To Corecursion

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It is! By terminating recursion on n. And the function defined from blueprint b turns out to satisfy the desired equation for f.

Eventual Up-To Corecursion

$\mathsf{f} : \mathbb{N} \to \mathsf{Str}$

$$f\,(n) = \begin{cases} \mathsf{zeros} & \text{if } n \leq 1 \\ f\,(n\,\mathsf{DIV}\,2) \odot (n \# f\,(3 * n + 1)) & \text{if } n > 1 \end{cases}$$

Standard blueprint synthesis fails:

- Friendly context around the calls
- Not all calls are guarded

We don't give up: Try to define a recursive blueprint. Is $b : \mathbb{N} \to \mathsf{GExpr}(\mathbb{N})$ is correctly defined?

$$b\,(n) = \begin{cases} \mathsf{Ct}\,(\mathsf{zeros}) & \text{if } n \leq 1 \\ b\,(n\,\mathsf{DIV}\,2)\ \boxed{\odot}\ (n\ \boxed{\#}\ \mathsf{Var}\,(3 * n + 1)) & \text{if } n > 1 \end{cases}$$

It is! By terminating recursion on n. And the function defined from blueprint b turns out to satisfy the desired equation for f.

Corecursion tool synthesizes equation for b
Recursive function package (Krauss et al.) validates this equation
Corecursion tool does the rest

Summary

Isabelle/HOL is one of the corecursion-friendliest proof assistants.
It supports:

- Functor-based (co)datatypes
- Incremental, up-to corecursion based on friendly operators
- Corresponding up-to coinduction (for equality)
- Mixed recursion-corecursion