4507/ 6507 Software and Hardware Verification
Introduction to LTL

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These slides contain material from Denisa Diaconescu, Georg Struth and Traian Florin Șerbănuță

LTL

LTL = Linear(-time) Temporal Logic

Introduced into computer science by Amir Pnueli in 1977

A logic for reasoning about execution paths of systems

One of the most important logics for software and hardware verification

Overview

Syntax: LTL formulas

Semantics: labeled transition systems

Practical specification patterns

Formula equivalence

Basic Intuition

- Consider execution paths of a system into the future.
- Label states with atomic propositions $p, q, r, \ldots$ that hold along paths at various points in time.
- LTL formulas can express regular patterns about these propositions as execution proceeds.

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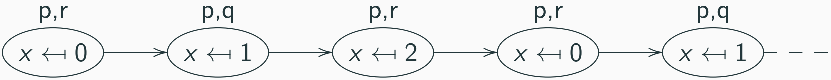

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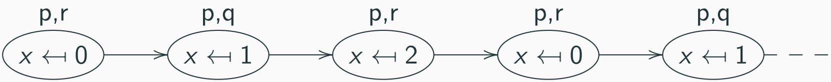

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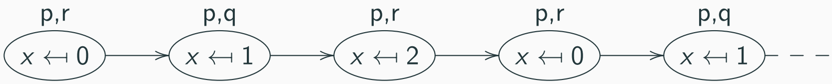

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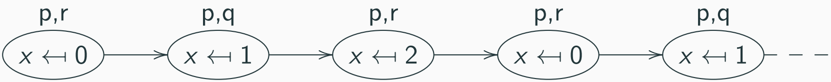

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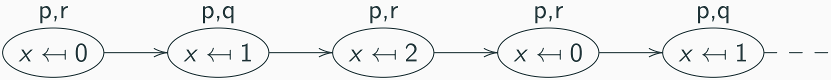

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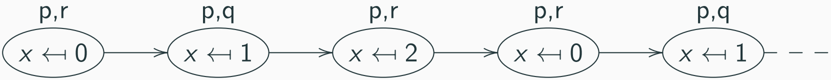

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Can you think of other patterns?

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- $\bigcirc\varphi$ – Next φ
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The unary connectives $\neg, \bigcirc, \Diamond, \Box$ have higher precedence than the binary connectives $\wedge, \vee, \rightarrow, \mathsf{U}$. E.g., $\Box\varphi \vee \psi$ is the same as $(\Box\varphi) \vee \psi$.

Syntax – Examples and Non-Examples

The following are LTL formulas:

- $(\Diamond p \wedge \Box q) \rightarrow (p \; \mathsf{U} \; r)$
- $\Diamond(p \rightarrow \Box r) \vee (\neg q \; \mathsf{U} \; p)$
- $p \; \mathsf{U} \; (q \; \mathsf{U} \; r)$
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Exercise. 1. Give five more examples of correctly constructed formulas. Include a formula that contains five atoms p, q, r, u, v, and a formula that contains three occurrences of $\Diamond$, one occurrence of $\Box$ and two occurrences of U. Read aloud the formulas that you have constructed.

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2. Give two examples of incorrectly constructed formulas that do not contain U or $\Box$.

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  - $\varphi \, \mathsf{U} \, \psi$ holds if φ holds until ψ holds; i.e., ψ holds now or at some point in the future, and φ holds continuously until then.

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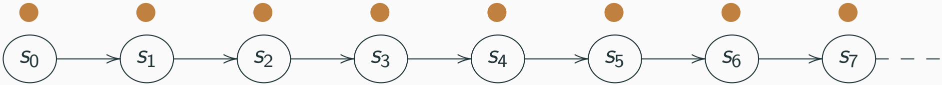

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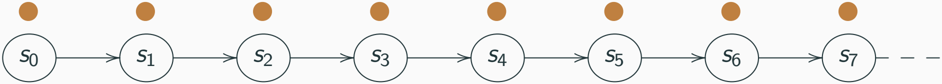
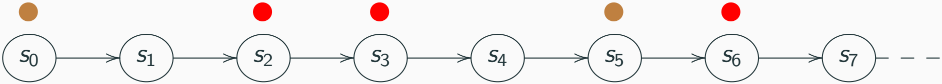


$\Box \neg(read \wedge write)$ means:

Always (i.e., now and at all points in the future), it is not the case that *read* and *write* hold. In other words: It is never the case that *read* and *write* hold at the same time.



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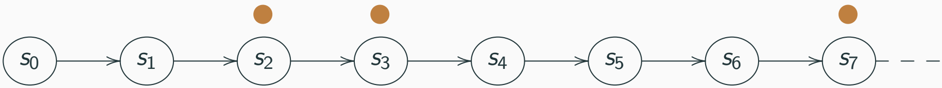

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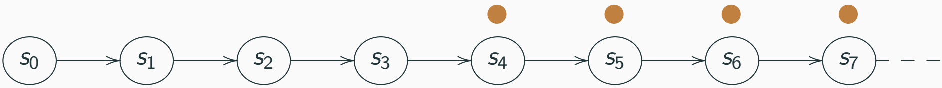

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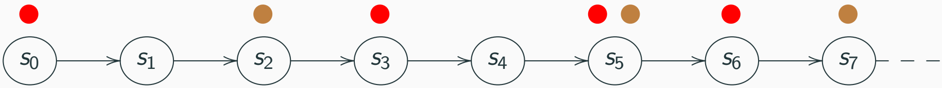
$\Box\,(request \rightarrow \Diamond grant)$ means:

Always [*request* implies eventually *grant*]. In other words: Always (i.e., now and at all points in the future), if *request* holds then eventually *grant* holds (i.e., there exists a point further up in the future where *grant* holds).



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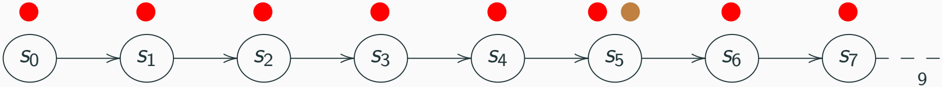
[Always *request*] implies [eventually *grant*]. In other words: If *request* holds at all points in time, then *grant* holds at some point in time.



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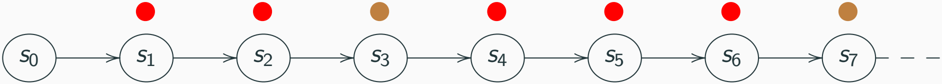
$\square\,(request \rightarrow (request \,\mathsf{U}\, grant))$ means:

Always, *request* implies [*request* until *grant*]. In other words: At every point in the future, if *request* holds than here exists a point further up in the future where *grant* holds, and *request* holds continuously until that point.



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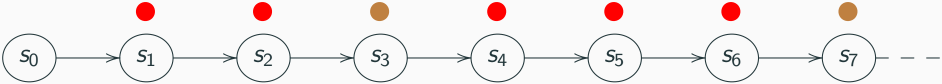

Exercise. Consider the following LTL formulas:

(a) $\Box\,(request\,\mathsf{U}\,grant)$

(b) $\Box\,\Diamond\,(request \rightarrow grant)$

(c) $\Box\,\Diamond\,request \rightarrow \Box\,\Diamond\,grant$

(d) $\Box\,\Diamond\,\Box\,enabled$

1. What is the correct way to parenthesize the point (c) formula, based on the operator precedence?

2. Depict graphically the meaning of these formulas. What is the difference between the point (d) formula and $\Diamond\,\Box\,enabled$?

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- A given process will be enabled infinitely often:

$$\square\lozenge \mathit{enabled}$$

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$$\Box\,(start \rightarrow active)$$

- It is always the case that requests are eventually granted:

$$\Box\,(request \rightarrow \Diamond grant)$$

- A given process will be enabled infinitely often:

$$\Box\Diamond enabled$$

- If a process is enabled infinitely often, then it will run infinitely often:

$$\Box\Diamond enabled \rightarrow \Box\Diamond run$$

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$$\Box(@2 \wedge \mathsf{upgoing} \wedge \mathsf{pressed5} \rightarrow (\mathsf{upgoing}\,\mathsf{U}\,@5))$$

Formal Semantics

Let S be a set of states and $L : S \to \mathcal{P}(Atoms)$ be a labeling function associating to each state s a set $L(s)$ of all atoms that are true in that state.
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$\pi \models_L \varphi \to \psi$	iff	$\pi \models_L \varphi$ implies $\pi \models_L \psi$

Formal Semantics

$$\pi \models_L \bigcirc \varphi \qquad \text{iff} \quad \pi^1 \models_L \varphi$$

Formal Semantics

$\pi \models_L \bigcirc\varphi$	iff	$\pi^1 \models_L \varphi$
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\pi \models_L \varphi \,\mathsf{U}\, \psi & \text{iff} & \text{there exists } i \geq 0 \text{ such that } \pi^i \models_L \psi \text{ and} \\
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\end{array}
$$

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$\models$ is called the satisfaction relation. It is a relation between formulas and infinite sequences of states in the presence of a state labeling with atom sets.

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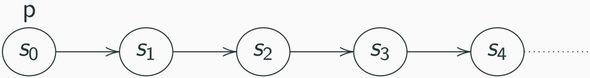
When the labeling L is fixed, we can write $\pi \models \varphi$ instead of $\pi \models_L \varphi$.

Semantics of Atoms Illustrated

$\pi \models p$



Semantics of "Next" Illustrated

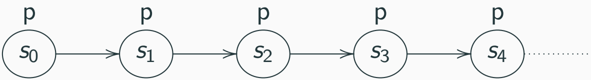
$\pi \models \bigcirc p$

Semantics of "Eventually" Illustrated

$\pi \models \lozenge p$

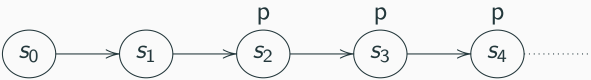
Semantics of "Always" Illustrated

$\pi \models \Box p$



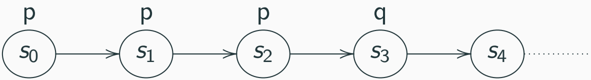
Combined Semantics of "Eventually" and "Always" Illustrated

$\pi \models \Diamond \Box p$



Semantics of "Until" Illustrated

$\pi \models p \, \mathsf{U} \, q$



Exercises

Transition Systems and Paths

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Paths are written as $\pi = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow \ldots$

Transition Systems and Paths – Example

Recall the example with two parallel processes, where, for $i \in \{1, 2\}$:

- n_i denotes "process i **n**ot in critical section"
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$Atoms = \{n_1, n_2, r_1, r_2, c_1, c_2\}$

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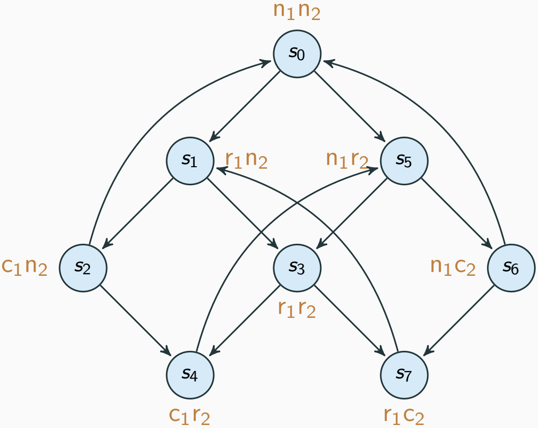

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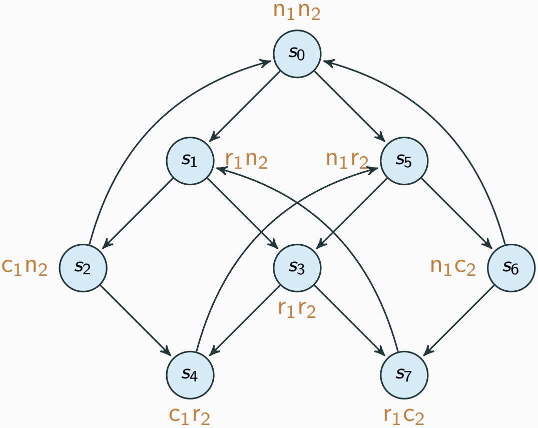

$\mathcal{M} = (S, \rightarrow, L)$ where

- $S = \{s_0, s_1, \ldots, s_7\}$
- $\rightarrow = \{(s_0, s_1), (s_0, s_5), \ldots\}$
- $L(s_0) = \{n_1, n_2\}$
- $L(s_1) = \{r_1, n_2\}$
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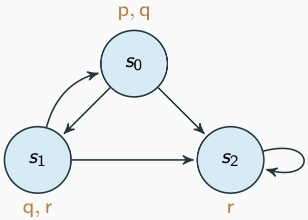

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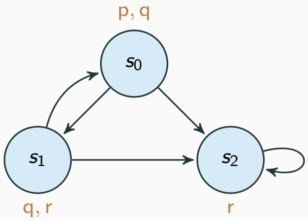

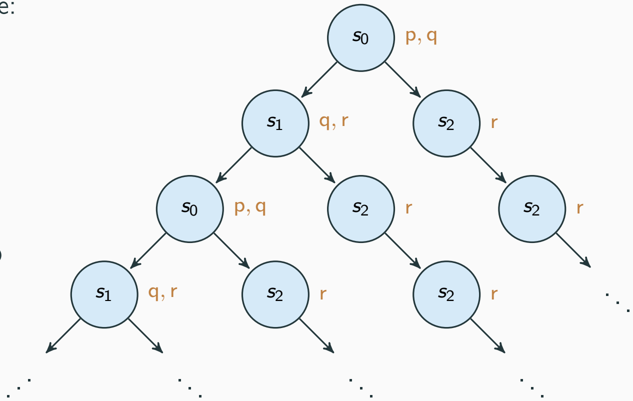

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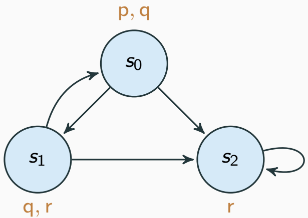

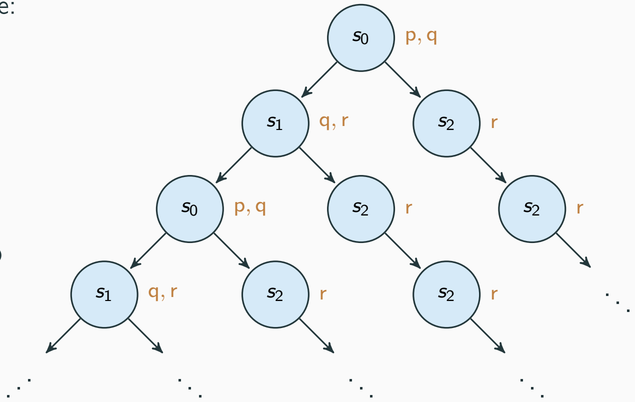

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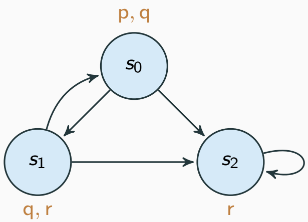

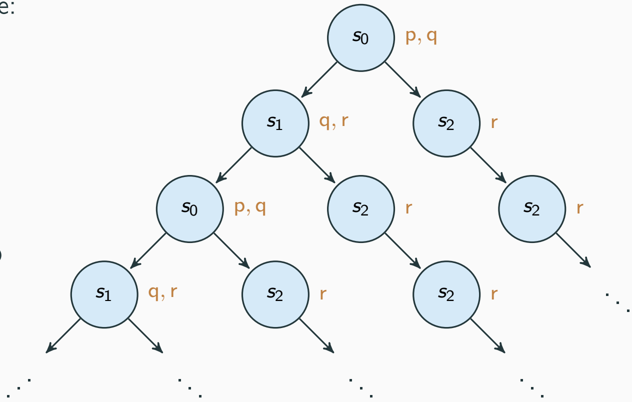

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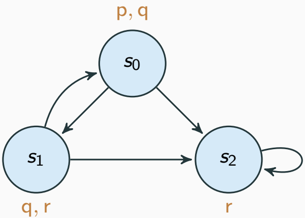

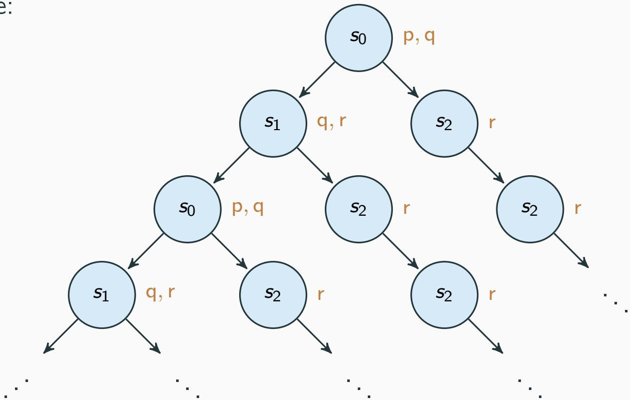

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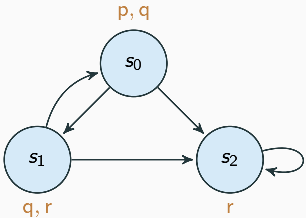

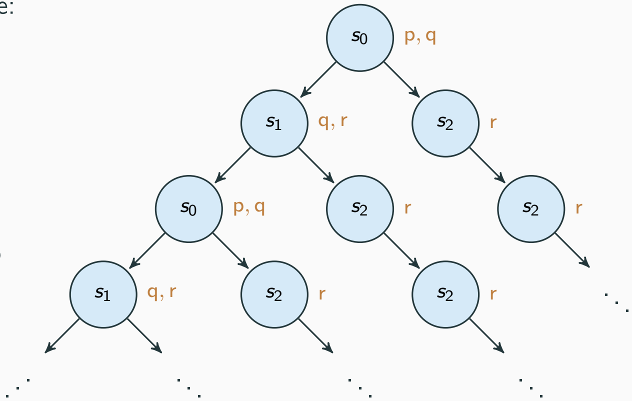

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$$(s_0 \to s_1 \to)^n \, s_0 \to (s_2 \to)^\infty \quad \text{for } n \geq 0$$

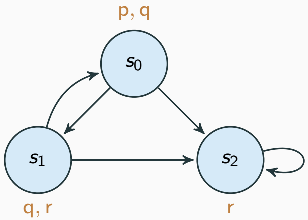
Formal Semantics Continued: Satisfaction Relation for LTSs

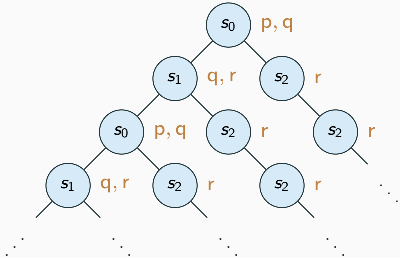
Let $\mathcal{M} = (S, \to, L)$ be an LTS and φ be an LTL formula.

We extend the satisfaction relation from infinite sequences to LTSs as follows:

For a state $s \in S$, we define $\mathcal{M}, s \models \varphi$, read $\mathcal{M}$ satisfies φ in state s or φ holds for $\mathcal{M}$ in state s, to mean that $\pi \models_L \varphi$ for every path π of $\mathcal{M}$ starting at state s.

Satisfaction Relation for LTSs – Example





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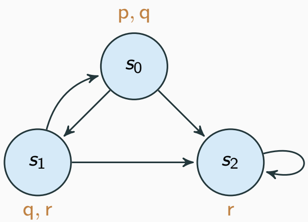
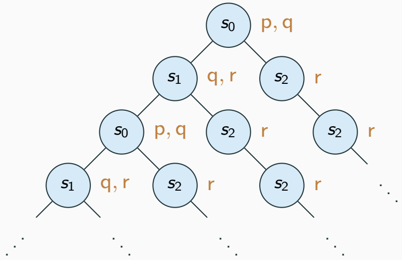
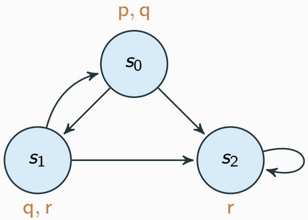
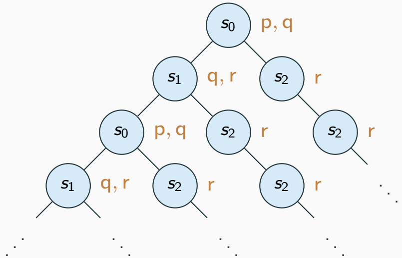




1. $\mathcal{M}, s_0 \models p \wedge q$

Satisfaction Relation for LTSs – Example





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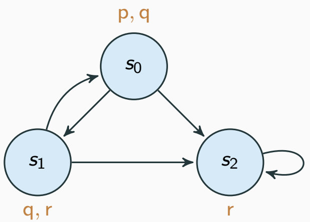

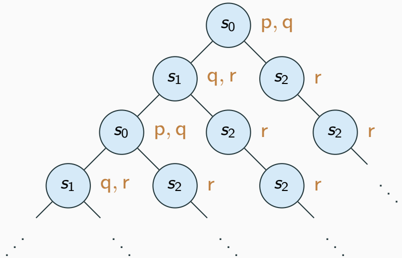

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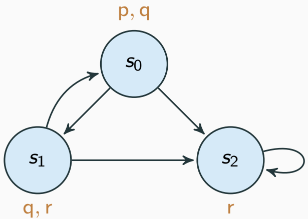
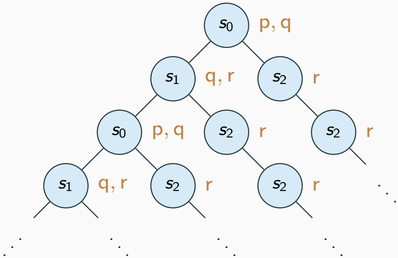
3. $\mathcal{M}, s_0 \models \bigcirc r$

Satisfaction Relation for LTSs – Example





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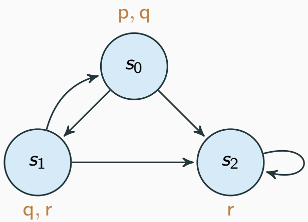
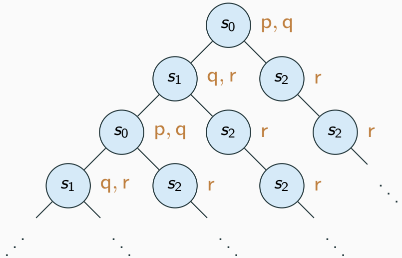
Satisfaction Relation for LTSs – Example





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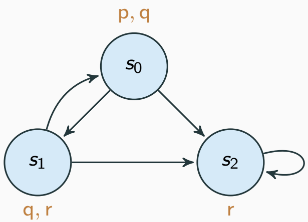

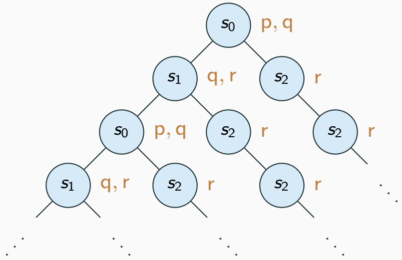

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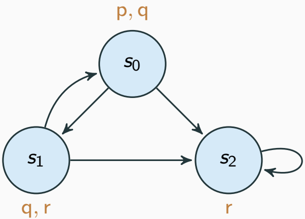
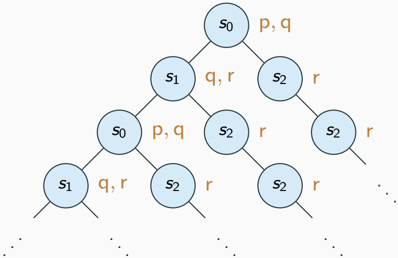
Satisfaction Relation for LTSs – Example





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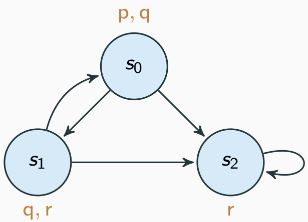
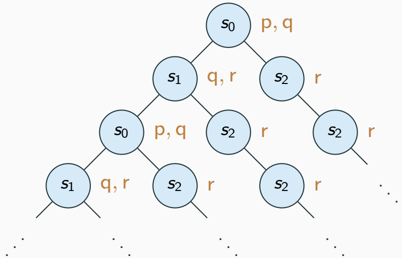
7. $\mathcal{M}, s_0 \models$ $\Diamond(\neg q \wedge r) \rightarrow \Diamond\Box r$

Satisfaction Relation for LTSs – Example





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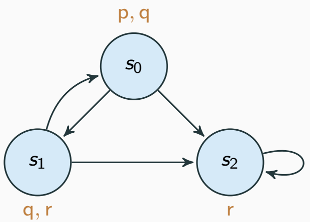
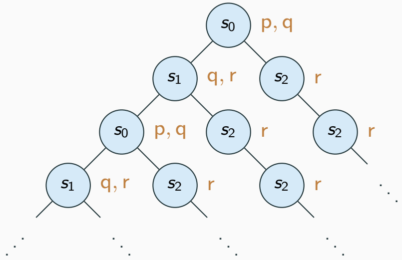
Satisfaction Relation for LTSs – Example





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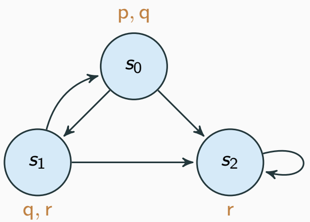
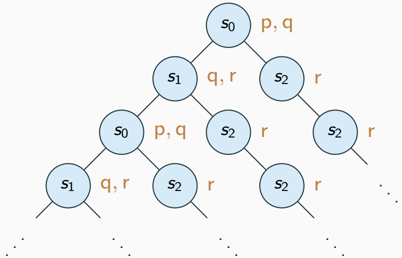
Satisfaction Relation for LTSs – Example





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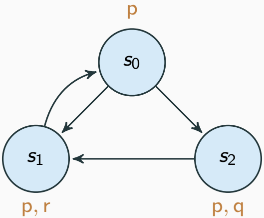
9. $\mathcal{M}, s_0 \models \Box \Diamond p \rightarrow \Box \Diamond r$
10. $\mathcal{M}, s_0 \not\models \Box \Diamond r \rightarrow \Box \Diamond p$

Homework Exercise 1

Consider the LTS shown in the picture:



1. Write down the mathematical definitions of its components S, $\to$ and L.
2. Draw its unwinding tree.
3. Describe all its possible paths that start at state s_0.
4. Determine which of the following are true, and explain why or why not:

$s_1 \models p \wedge r$ $\qquad$ $s_0 \models \bigcirc r$

$s_0 \models \bigcirc(p \vee r)$ $\qquad$ $s_2 \models \Box p$

$s_0 \models (p \vee q) \,\mathrm{U}\, r$ $\qquad$ $s_1 \models (p \wedge \neg r) \,\mathrm{U}\, q$

5. Give your own examples of LTL formulas and states such that the formula holds or does not hold in the given state, and in each case explain why.

Homework Exercise 2

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold.

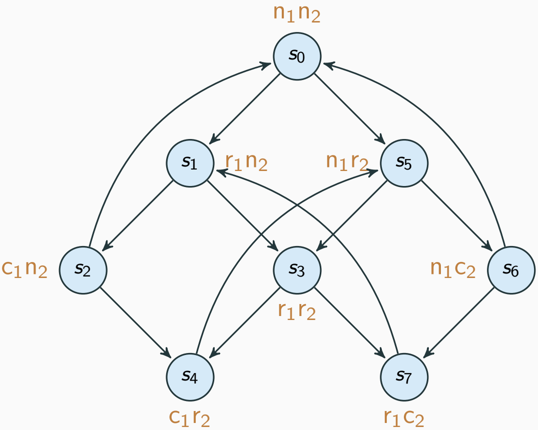
- The safety property: Only one process may execute critical section code at any point
- The liveness property: Whenever a process requests to enter its critical section, it will eventually be allowed to do so.
- The non-blocking property: A process can always request to enter its critical section.

Transition Systems and Paths – Example

Recall the example with two parallel processes, where, for $i \in \{1, 2\}$:

- n_i denotes "process i **n**ot in critical section"
- r_i denotes "process i **r**equesting to enter critical section"
- c_i denotes "process i in **c**ritical section"

$Atoms = \{n_1, n_2, r_1, r_2, c_1, c_2\}$



$\mathcal{M} = (S, \rightarrow, L)$ where

- $S = \{s_0, s_1, \ldots, s_7\}$
- $\rightarrow = \{(s_0, s_1), (s_0, s_5), \ldots\}$
- $L(s_0) = \{n_1, n_2\}$
- $L(s_1) = \{r_1, n_2\}$
- ...

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

An LTL formula expressing this is $\varphi = \Box\,(\neg(c_1 \wedge c_2))$.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \to, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

An LTL formula expressing this is $\varphi = \Box(\neg(c_1 \wedge c_2))$. Let's prove that $\mathcal{M}, s_0 \models \varphi$.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

An LTL formula expressing this is $\varphi = \square\,(\neg(c_1 \wedge c_2))$. Let's prove that $\mathcal{M}, s_0 \models \varphi$.

$\mathcal{M}, s_0 \models \varphi$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).
Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

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An LTL formula expressing this is $\varphi = \Box(\neg(c_1 \wedge c_2))$. Let's prove that $\mathcal{M}, s_0 \models \varphi$.

$\mathcal{M}, s_0 \models \varphi$
means (by the semantics in an LTS)
for all $\pi \in Paths_{s_0}(\mathcal{M})$, $\pi \models_L \varphi$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

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$\mathcal{M}, s_0 \models \varphi$
means (by the semantics in an LTS)
for all $\pi \in Paths_{s_0}(\mathcal{M})$, $\pi \models_L \varphi$
which means (by the semantics of $\Box$)
for all $\pi \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, $\pi^i \models_L \neg(c_1 \wedge c_2)$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).
Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

An LTL formula expressing this is $\varphi = \Box(\neg(c_1 \land c_2))$. Let's prove that $\mathcal{M}, s_0 \models \varphi$.

$\mathcal{M}, s_0 \models \varphi$
means (by the semantics in an LTS)
for all $\pi \in Paths_{s_0}(\mathcal{M}), \ \pi \models_L \varphi$
which means (by the semantics of $\Box$)
for all $\pi \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, $\pi^i \models_L \neg(c_1 \land c_2)$
which means (by the semantics of the propositional connectives and atoms)
for all $\pi = t_0 t_1 t_2 \ldots \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, not $(c_1 \in L(t_i)$ and $c_2 \in L(t_i))$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0). Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

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which means (by the semantics of the propositional connectives and atoms)
for all $\pi = t_0 t_1 t_2 \ldots \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, not ($c_1 \in L(t_i)$ and $c_2 \in L(t_i)$)
which is implied by
for all $s \in S$, not ($c_1 \in L(s)$ and $c_2 \in L(s)$)

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).
Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

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which is implied by
for all $s \in S$, not $(c_1 \in L(s)$ and $c_2 \in L(s))$
which is true – can be checked by inspecting the system.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).
Let $\mathcal{M} = (S, \to, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

An LTL formula expressing this is $\varphi = \Box(\neg(c_1 \wedge c_2))$. Let's prove that $\mathcal{M}, s_0 \models \varphi$.

$\mathcal{M}, s_0 \models \varphi$
means (by the semantics in an LTS)
for all $\pi \in Paths_{s_0}(\mathcal{M})$, $\pi \models_L \varphi$
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for all $\pi \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, $\pi^i \models_L \neg(c_1 \wedge c_2)$
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for all $\pi = t_0 t_1 t_2 \ldots \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, not ($c_1 \in L(t_i)$ and $c_2 \in L(t_i)$)
which is implied by
for all $s \in S$, not ($c_1 \in L(s)$ and $c_2 \in L(s)$)
which is true – can be checked by inspecting the system.

We conclude that $\mathcal{M}, s_0 \models \varphi$.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

Safety property: Only one process may execute critical section code at any point.

An LTL formula expressing this is $\varphi = \Box(\neg(c_1 \wedge c_2))$. Let's prove that $\mathcal{M}, s_0 \models \varphi$.

$\mathcal{M}, s_0 \models \varphi$

means (by the semantics in an LTS)

for all $\pi \in Paths_{s_0}(\mathcal{M})$, $\pi \models_L \varphi$

which means (by the semantics of $\Box$)

for all $\pi \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, $\pi^i \models_L \neg(c_1 \wedge c_2)$

which means (by the semantics of the propositional connectives and atoms)

for all $\pi = t_0 t_1 t_2 \ldots \in Paths_{s_0}(\mathcal{M})$, for all $i \geq 0$, not $(c_1 \in L(t_i)$ and $c_2 \in L(t_i))$

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for all $s \in S$, not $(c_1 \in L(s)$ and $c_2 \in L(s))$

which is true – can be checked by inspecting the system.

We conclude that $\mathcal{M}, s_0 \models \varphi$.

This was backwards reasoning, reducing the goal to something true.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

The liveness property: Whenever a process requests to enter its critical section, it will eventually be allowed to do so.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

The liveness property: Whenever a process requests to enter its critical section, it will eventually be allowed to do so.

An LTL formula expressing this is $\varphi = \Box\,((r_1 \rightarrow \Diamond c_1) \land (r_2 \rightarrow \Diamond c_2))$.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

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An LTL formula expressing this is $\varphi = \Box\,((r_1 \rightarrow \Diamond c_1) \wedge (r_2 \rightarrow \Diamond c_2))$.

Let's prove that $\mathcal{M}, s_0 \not\models \varphi$.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

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By the semantics in an LTS, it suffices to find one $\pi \in Path_{s_0}(\mathcal{M})$ such that $\pi \not\models_L \varphi$.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

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By the semantics in an LTS, it suffices to find one $\pi \in Path_{s_0}(\mathcal{M})$ such that $\pi \not\models_L \varphi$.

We take $\pi = s_0(s_1 s_3 s_7)^\infty$.

$\pi \models_L \varphi$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).
Let $\mathcal{M} = (S, \to, L)$ be that transition system.

The liveness property: Whenever a process requests to enter its critical section, it will eventually be allowed to do so.
An LTL formula expressing this is $\varphi = \Box((r_1 \to \Diamond c_1) \land (r_2 \to \Diamond c_2))$.
Let's prove that $\mathcal{M}, s_0 \not\models \varphi$.
By the semantics in an LTS, it suffices to find one $\pi \in Path_{s_0}(\mathcal{M})$ such that $\pi \not\models_L \varphi$.
We take $\pi = s_0(s_1 s_3 s_7)^\infty$.

$\pi \models_L \varphi$
implies, by the semantics of $\Box$ and $\land$
$(s_1 s_3 s_7)^\infty \models_L r_1 \to \Diamond c_1$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

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$\pi \models_L \varphi$

implies, by the semantics of $\Box$ and $\wedge$

$(s_1 s_3 s_7)^\infty \models_L r_1 \rightarrow \Diamond c_1$

which implies, by the semantics of $\rightarrow$ and atoms (since $r_1 \in L(s_1)$)

$(s_1 s_3 s_7)^\infty \models_L \Diamond c_1$

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In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

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Let's prove that $\mathcal{M}, s_0 \not\models \varphi$.

By the semantics in an LTS, it suffices to find one $\pi \in Path_{s_0}(\mathcal{M})$ such that $\pi \not\models_L \varphi$.

We take $\pi = s_0(s_1 s_3 s_7)^\infty$.

$\pi \models_L \varphi$

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which implies, by the semantics of $\rightarrow$ and atoms (since $r_1 \in L(s_1)$)

$(s_1 s_3 s_7)^\infty \models_L \Diamond c_1$

which implies, by the semantics of $\Diamond$ and atoms

$c_1 \in L(s_1)$ or $c_1 \in L(s_3)$ or $c_1 \in L(s_7)$

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).

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By the semantics in an LTS, it suffices to find one $\pi \in Path_{s_0}(\mathcal{M})$ such that $\pi \not\models_L \varphi$.

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implies, by the semantics of $\Box$ and $\wedge$

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which implies, by the semantics of $\rightarrow$ and atoms (since $r_1 \in L(s_1)$)

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which is false – as can be seen by inspecting the system.

Homework Exercise 2 – Solution

In the example with the two processes executed in parallel, determine whether the following properties are expressible in LTL; and if yes, whether they hold (for s_0).
Let $\mathcal{M} = (S, \rightarrow, L)$ be that transition system.

The liveness property: Whenever a process requests to enter its critical section, it will eventually be allowed to do so.
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Since the assumption $\pi \models_L \varphi$ leads to a contradiction, we conclude $\pi \not\models_L \varphi$.

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The non-blocking property: A process can always request to enter its critical section (provided it is not there already).

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The properties NB1 and NB2 (hence NB as well) are true about the system $\mathcal{M}$.
This can be routinely checked by:
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But NB1, NB2 and NB are not expressible as LTL formulas.
Can we prove this? Hmm... what does it even mean?

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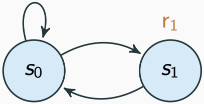

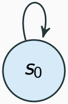

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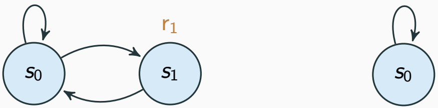

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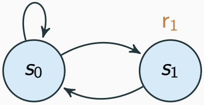

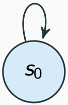

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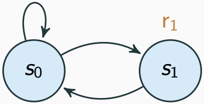

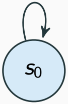

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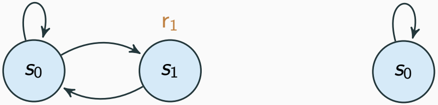

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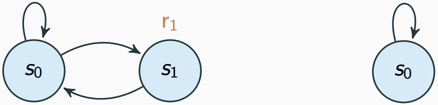

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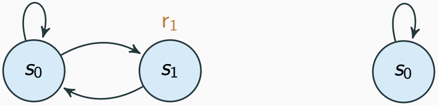

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Homework: Modify the proof to show that NB is not expressible in LTL.

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Two formulas φ and ψ are equivalent, denoted $\varphi \equiv \psi$, if they are satisfied by (i.e., hold for) exactly the same state labelings and infinite sequences of states: Given any labeling $L : S \to \mathcal{P}(Atoms)$ and any infinite sequence of states π, we have that $\pi \models_L \varphi$ iff $\pi \models_L \psi$

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Homework Exercise 3: Explain why this is the case.

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Propositional tautologies:

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Expansion laws:

$$\Diamond\varphi \equiv \varphi \vee \bigcirc\Diamond\varphi \qquad \Box\varphi \equiv \varphi \wedge \bigcirc\Box\varphi \qquad \varphi\,\mathsf{U}\,\psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi\,\mathsf{U}\,\psi))$$

Proving Formula Equivalences

Let us prove the following equivalence:

$$\Diamond\varphi \equiv \neg\Box\neg\varphi$$

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Proving Formula Equivalences

Let us prove the following equivalence:

$$\Diamond \varphi \equiv \neg \Box \neg \varphi$$

Fix a labeling function $L : S \to \mathcal{P}(Atoms)$ and let π be an infinite sequence $s_0 s_1 s_2 \ldots$. We must prove two things:

(1) $\pi \models \Diamond \varphi$ implies $\pi \models \neg \Box \neg \varphi$.

(2) $\pi \models \neg \Box \neg \varphi$ implies $\pi \models \Diamond \varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Hence, by semantics of $\Diamond$, there exists an i such that $\pi^i \models \varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Hence, by semantics of $\Diamond$, there exists an i such that $\pi^i \models \varphi$.

Hence, by logic, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Hence, by semantics of $\Diamond$, there exists an i such that $\pi^i \models \varphi$.

Hence, by logic, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Hence, by semantics of $\neg$, it is not the case that: for all i, $\pi^i \models \neg\varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Hence, by semantics of $\Diamond$, there exists an i such that $\pi^i \models \varphi$.

Hence, by logic, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Hence, by semantics of $\neg$, it is not the case that: for all i, $\pi^i \models \neg\varphi$.

Hence, by semantics of $\Box$, it is not the case that $\pi \models \Box\neg\varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Hence, by semantics of $\Diamond$, there exists an i such that $\pi^i \models \varphi$.

Hence, by logic, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Hence, by semantics of $\neg$, it is not the case that: for all i, $\pi^i \models \neg\varphi$.

Hence, by semantics of $\Box$, it is not the case that $\pi \models \Box\neg\varphi$.

In other words, $\pi \not\models \Box\neg\varphi$.

Proving Formula Equivalences

Proving that $\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$:

Assume $\pi \models \Diamond\varphi$.

Hence, by semantics of $\Diamond$, there exists an i such that $\pi^i \models \varphi$.

Hence, by logic, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Hence, by semantics of $\neg$, it is not the case that: for all i, $\pi^i \models \neg\varphi$.

Hence, by semantics of $\Box$, it is not the case that $\pi \models \Box\neg\varphi$.

In other words, $\pi \not\models \Box\neg\varphi$.

Hence, by semantics of $\neg$, we have $\pi \models \neg\Box\neg\varphi$.

Proving Formula Equivalences

Proving that $\pi \models \neg \Box \neg \varphi$ implies $\pi \models \Diamond \varphi$:

Proving Formula Equivalences

Proving that $\pi \models \neg \Box \neg \varphi$ implies $\pi \models \Diamond \varphi$:

Assume $\pi \models \neg \Box \neg \varphi$.

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Hence, by semantics of $\neg$, we have $\pi \not\models \Box \neg \varphi$.

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Hence, by semantics of $\Box$, it is not the case that: for all i, $\pi^i \models \neg \varphi$.

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Proving Formula Equivalences

Proving that $\pi \models \neg \Box \neg \varphi$ implies $\pi \models \Diamond \varphi$:

Assume $\pi \models \neg \Box \neg \varphi$.

Hence, by semantics of $\neg$, we have $\pi \not\models \Box \neg \varphi$.

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Hence, by semantics of $\Box$, it is not the case that: for all i, $\pi^i \models \neg \varphi$.

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Proving that $\pi \models \neg \Box \neg \varphi$ implies $\pi \models \Diamond \varphi$:

Assume $\pi \models \neg \Box \neg \varphi$.

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Hence, by semantics of $\Box$, it is not the case that: for all i, $\pi^i \models \neg \varphi$.

Hence, by semantics of $\neg$, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Hence, by logic, there exists an i such that $\pi^i \models \varphi$.

Hence, by semantics of $\Diamond$, we have $\pi \models \Diamond \varphi$.

Proving Formula Equivalences

Proving that $\pi \models \neg \Box \neg \varphi$ implies $\pi \models \Diamond \varphi$:

Assume $\pi \models \neg \Box \neg \varphi$.

Hence, by semantics of $\neg$, we have $\pi \not\models \Box \neg \varphi$.

In other words, it is not the case that $\pi \models \Box \neg \varphi$.

Hence, by semantics of $\Box$, it is not the case that: for all i, $\pi^i \models \neg \varphi$.

Hence, by semantics of $\neg$, it is not the case that: for all i, $\pi^i \not\models \varphi$.

Hence, by logic, there exists an i such that $\pi^i \models \varphi$.

Hence, by semantics of $\Diamond$, we have $\pi \models \Diamond \varphi$.

Note. The proof of "$\pi \models \neg\Box\neg\varphi$ implies $\pi \models \Diamond\varphi$" is the reverse of the proof of "$\pi \models \Diamond\varphi$ implies $\pi \models \neg\Box\neg\varphi$". So we could have proved directly "$\pi \models \Diamond\varphi$ iff $\pi \models \neg\Box\neg\varphi$" by a chain of equivalent (iff-related) statements.

Homework Exercise 4

Choose from the previous two slides any three laws (except for the propositional tautologies) and prove them.

Hint. Take the approach shown above, using the semantics of formulas and logical reasoning.

Summary of the Discussed Concepts

- LTL = Linear Temporal Logic
- Syntax = formulas built from
  - atoms
  - propositional connectives
  - temporal connectives
- LTL can express some practical specification patterns
- Semantics = the satisfaction relation
  - between infinite sequences and formulas
  - between LTSs and formulas
- Formula equivalence

Further Reading

Sections 5.1.1–5.1.4 of Baier & Katoen's "Principles of Model Checking" (MIT Press 2008)

Section 3.2 of Huth & Ryan's "Logic in Computer Science: Modelling and Reasoning about Systems" (Cambridge University Press 2004)

Note. Uses another (standard) notation for the temporal connectives:

- X instead of $\bigcirc$
- F instead of $\Diamond$ (think "in the Future")
- G instead of $\Box$ (think "Globally")