4507/6507 Software and Hardware Verification
Computation Tree Logic (CTL)

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These slides contain material from Denisa Diaconescu and Traian Florin Șerbănuță

Introduction

CTL

CTL = Computation Tree Logic

Introduced by Edmund M. Clarke and E. Allen Emerson in 1981

A temporal logic for reasoning about transition systems

An alternative to LTL

Syntax

CTL Syntax – Definition

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$$\forall\bigcirc\varphi \mid \exists\bigcirc\varphi \mid \forall\Diamond\varphi \mid \exists\Diamond\varphi \mid \forall\Box\varphi \mid \exists\Box\varphi \mid \varphi\,\forall\mathsf{U}\,\psi \mid \varphi\,\exists\mathsf{U}\,\psi$$

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Each temporal connective is a pair of a path quantifier:

- $\forall$ – for all paths
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Precedence: As usual, unary connectives bind more strongly than binary ones.

CTL Syntax – Examples

The following are CTL formulas:

$$\forall\Box\,(b \rightarrow \exists\Box\, c)$$

$$(\exists\Diamond\, a)\, \exists\mathsf{U}\, b$$

$$a\, \forall\mathsf{U}\, (\exists\Diamond\, b)$$

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$$(\exists\Diamond\,\exists\Box\, a) \to (\forall\Diamond\, b)$$

The following are not CTL formulas:

$$\exists\Diamond\,\Box b$$

$$\forall\neg\Box\neg a$$

$$\Diamond(a\,\mathsf{U}\, b)$$

$$\exists\Diamond\,(a\,\mathsf{U}\, b)$$

Semantics

Labeled Transition Systems and Paths Recalled

(These are the same as for LTL.)

A labeled transition system (LTS for short) is a triple $\mathcal{M} = (S, \rightarrow, L)$ consisting of:

- S a finite set of states
- $\rightarrow \subseteq S \times S$ a transition relation
- $L : S \rightarrow \mathcal{P}(Atoms)$ a labeling function

such that every state has an outward transition, i.e., for all $s_1 \in S$ there exists $s_2 \in S$ with $s_1 \rightarrow s_2$.

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A path π in an LTS $\mathcal{M} = (S, \rightarrow, L)$ is an infinite sequence of states $s_0 s_1 s_2 \ldots$ such that for all $i \geq 0$, $s_i \rightarrow s_{i+1}$.

Given $s \in S$, we write $Paths_s(\mathcal{M})$ for the set of all paths in $\mathcal{M}$ that start from s.

Towards Defining the CTL Semantics

Recall:

- LTL satisfaction is first defined on linear structures, i.e., on infinite sequences of states $\pi = s_0 s_1 \ldots$ (given $L : S \to \mathcal{P}(Atoms)$): $\pi \models_L \varphi$

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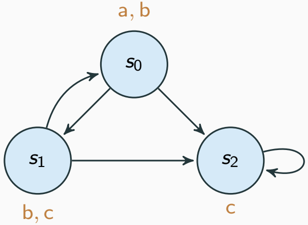
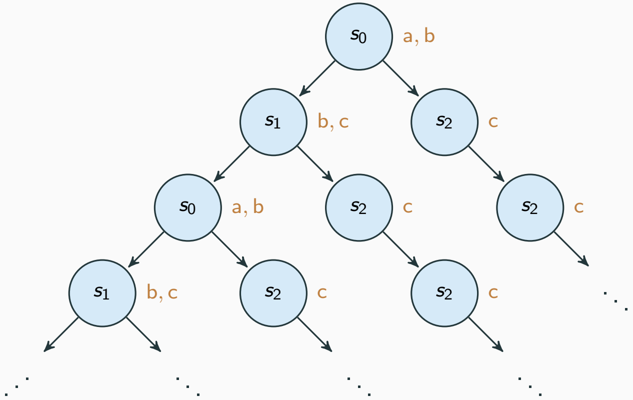
By contrast, for CTL:

- satisfaction will be defined directly on the branching structures, i.e., on LTSs
- … and it is most intuitive to think in terms of the unwinding tree, a.k.a. computation tree, of an LTS

Recall: The Unwinding (Computation) Tree of an LTS

An LTS (on the left) and its unwinding tree starting in s_0 (on the right):





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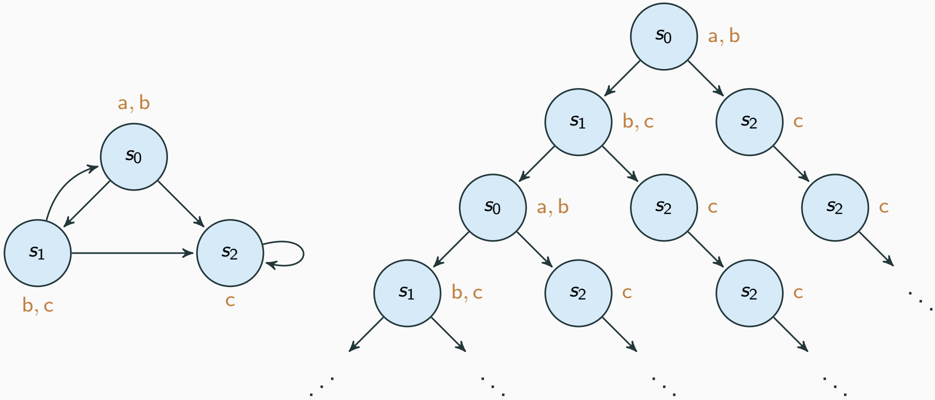

Note: The LTS and its unwinding tree have the same paths.

CTL Semantics in Pictures

In the following illustrations:

- we consider specific CTL formulas
- we draw part of a sample unwinding tree of an LTS
- using color coding when necessary, we highlight states where subformulas of the formula are supposed to hold according to the intended semantics

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- by "a current or future state" we mean "the current state or a future state"
- by "all current and future states" we mean "the current state and all the future states"

CTL Semantics in Pictures

"For All Eventually"

$$\forall \Diamond \varphi$$

For all paths, φ eventually holds.

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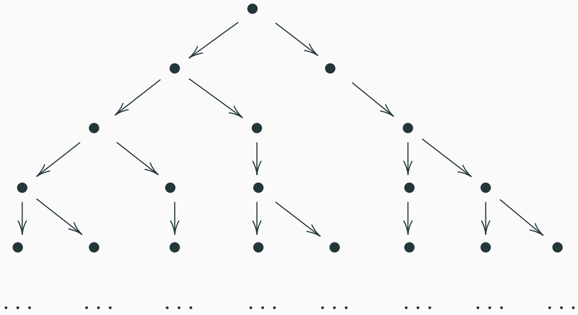

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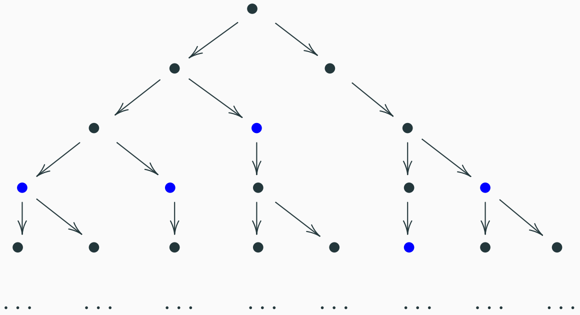

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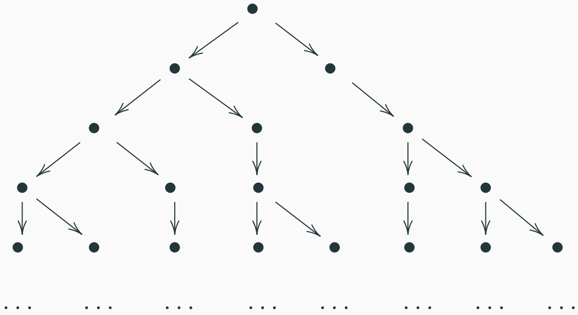

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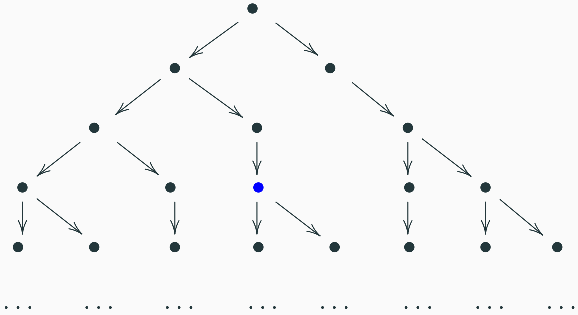

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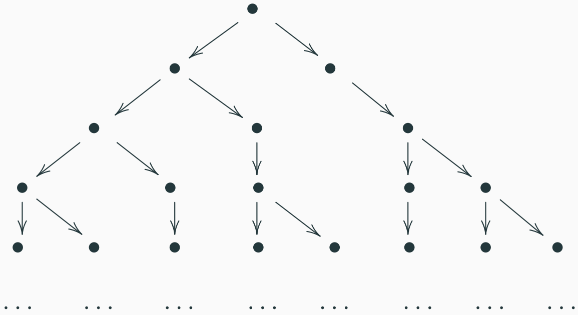

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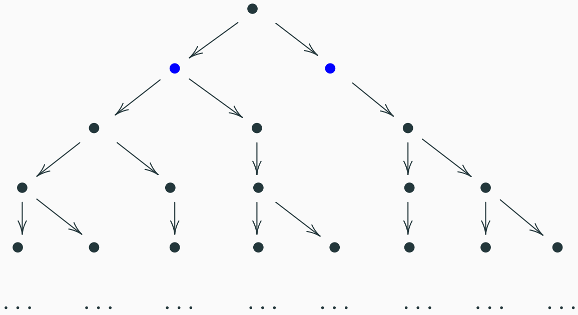

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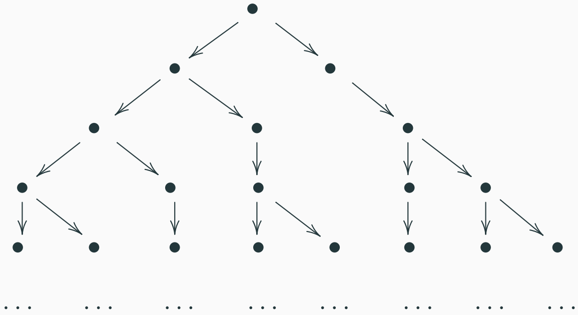

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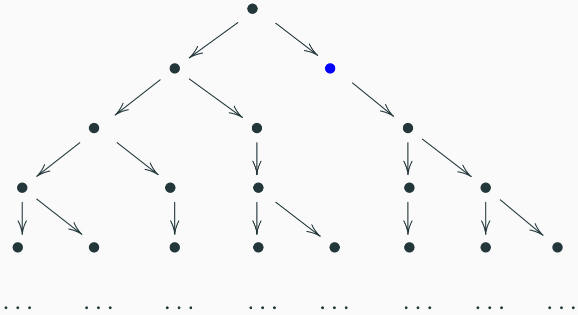

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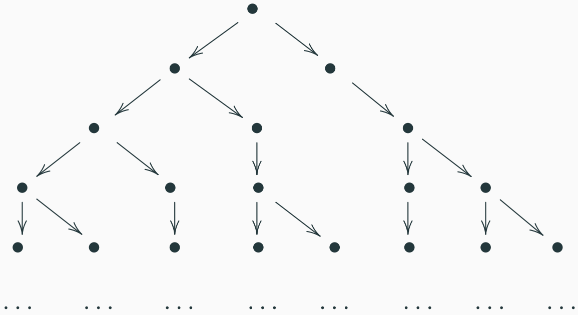

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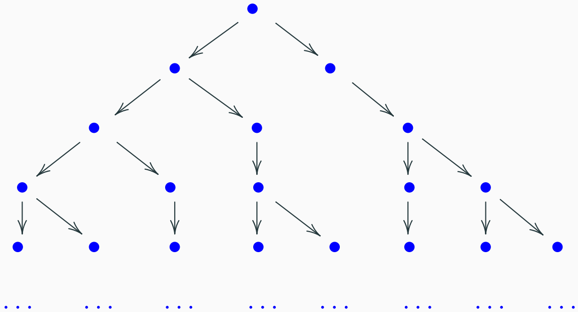

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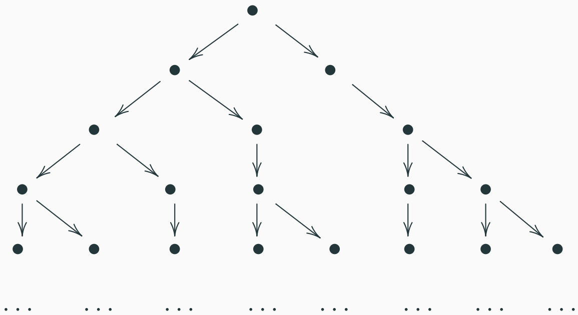

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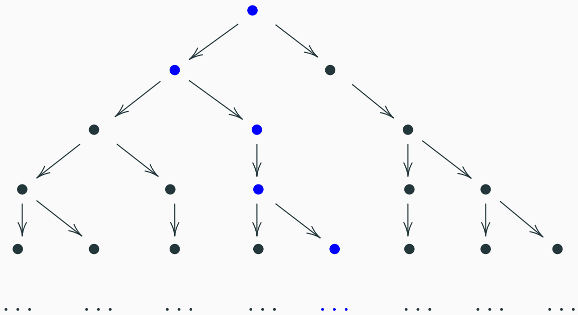

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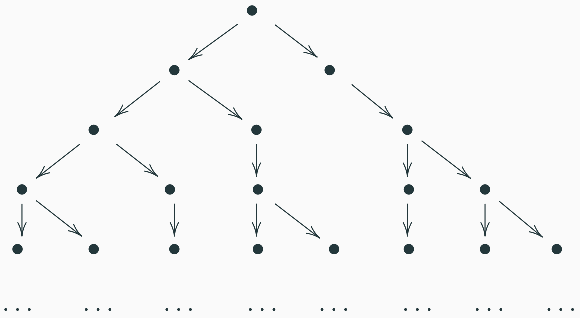

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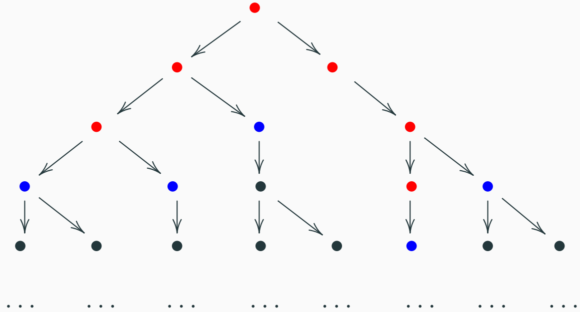

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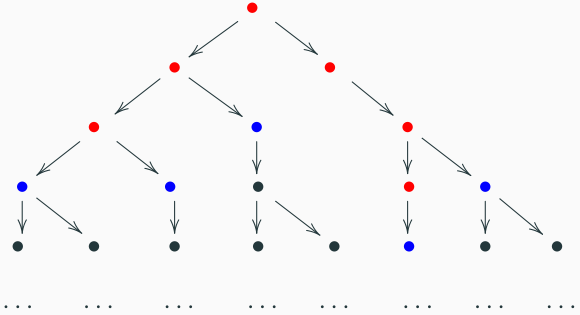

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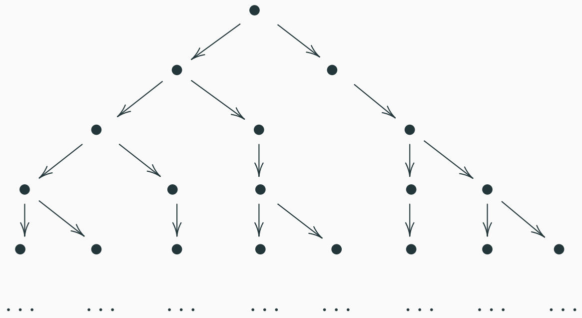

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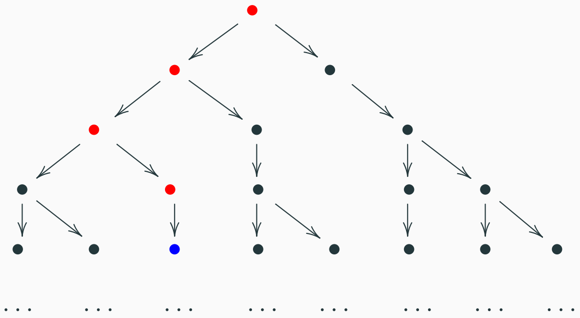

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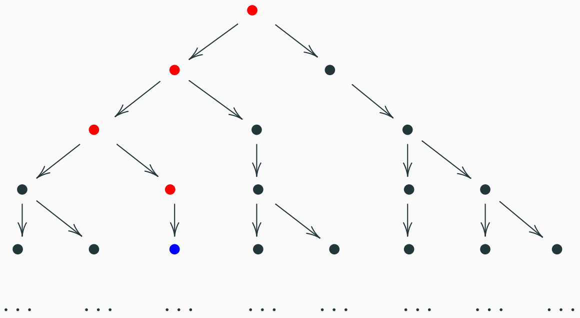

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CTL Semantics in Pictures – Combining Connectives

In what follows:

- "further up in all possible futures" will mean
  "on the current state and on all future states from there"
- "further up in a possible future" will mean
  "on the current state or on some future state form there"

CTL Semantics in Pictures – Combining Connectives

"For All Always" followed by "There Exists Eventually"

$\forall\Box\exists\Diamond\varphi$

for all current and future states,

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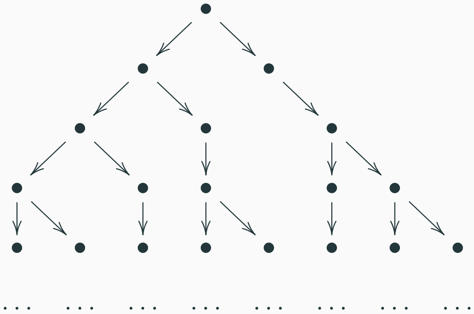

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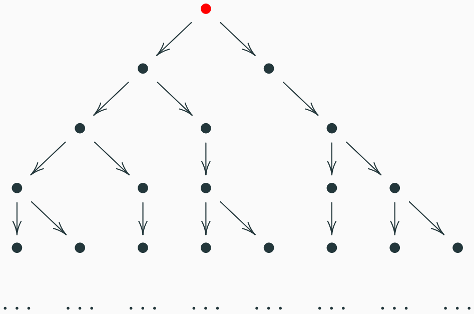

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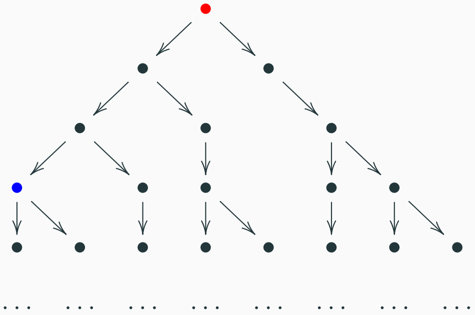

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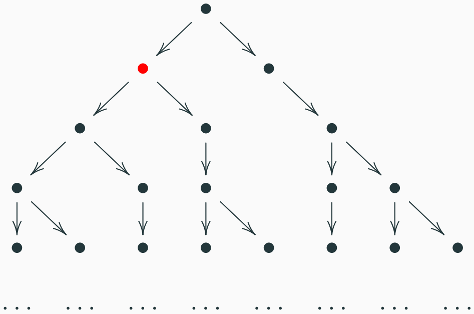

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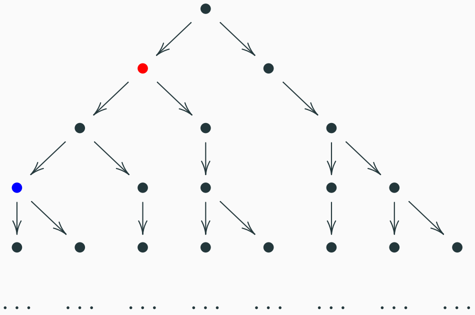

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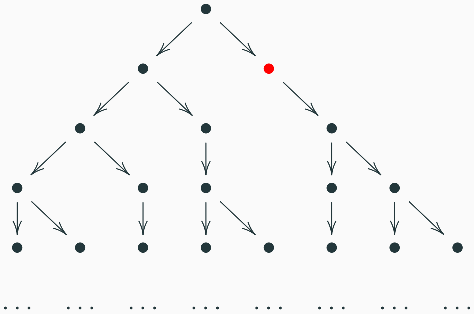

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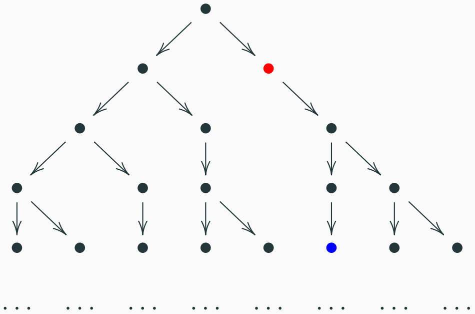

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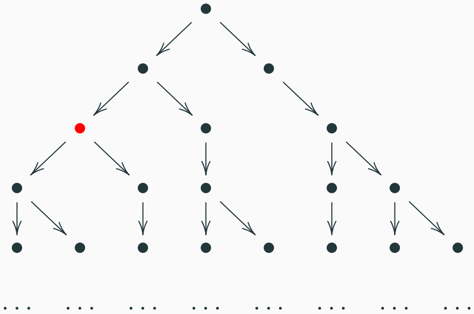

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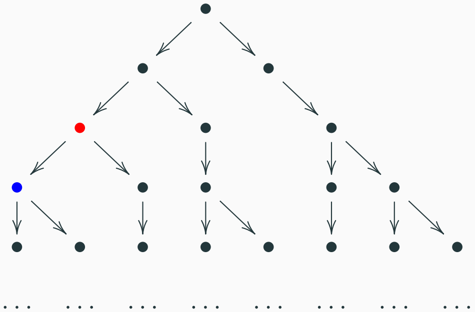

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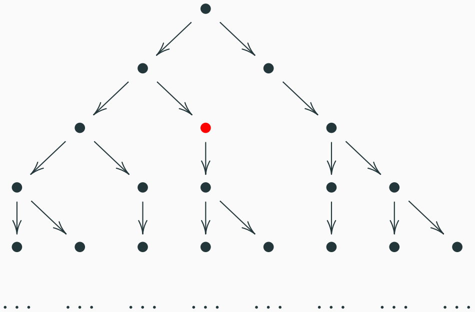

CTL Semantics in Pictures – Combining Connectives

"For All Always" followed by "There Exists Eventually"

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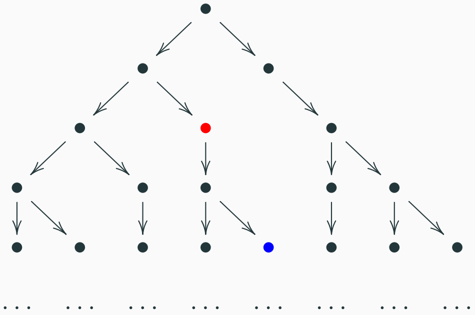

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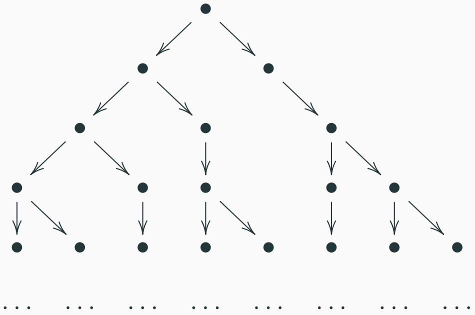

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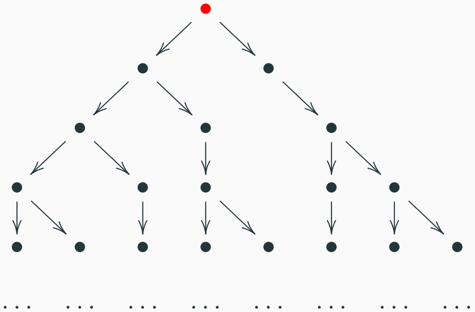

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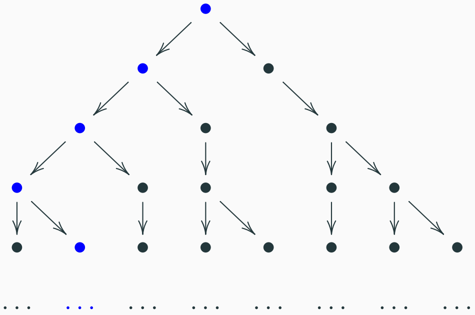

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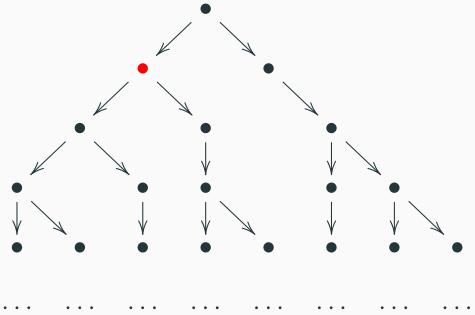

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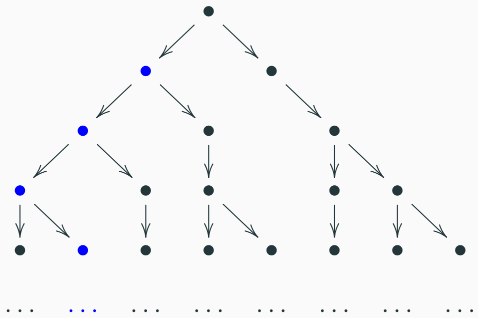

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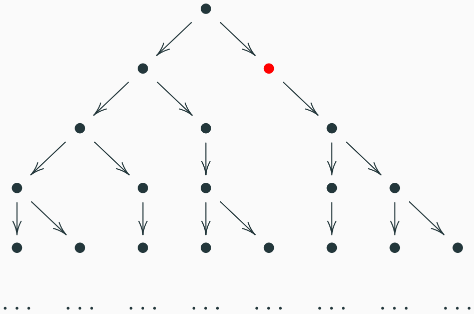

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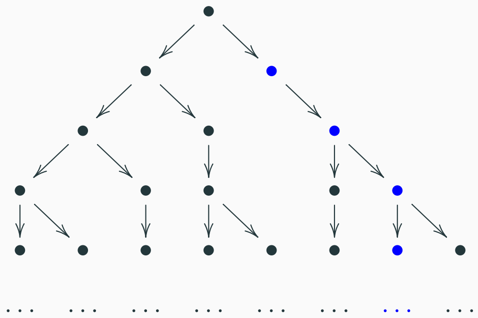

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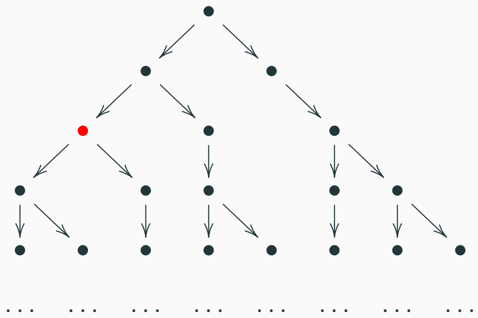

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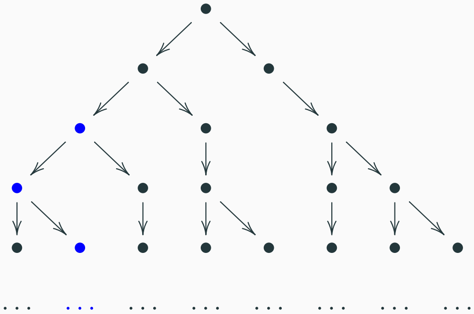

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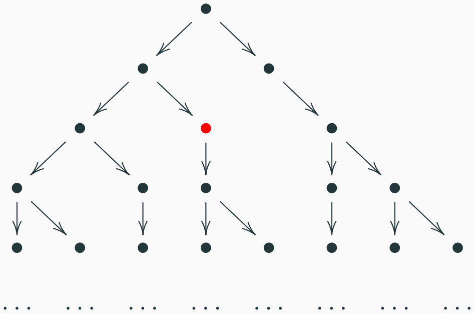

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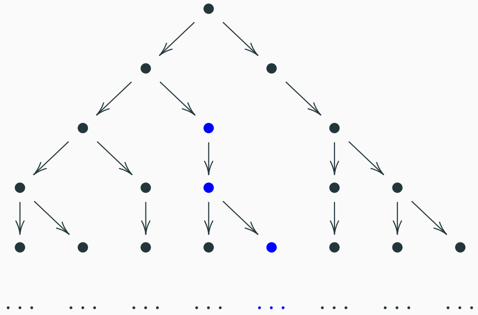

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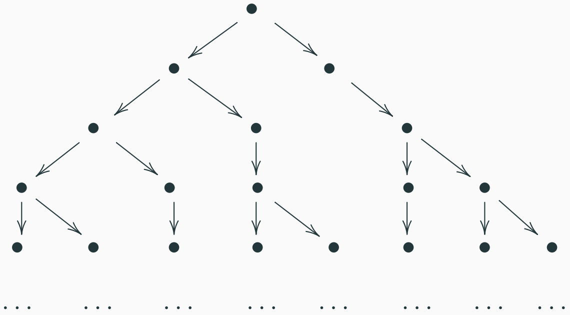

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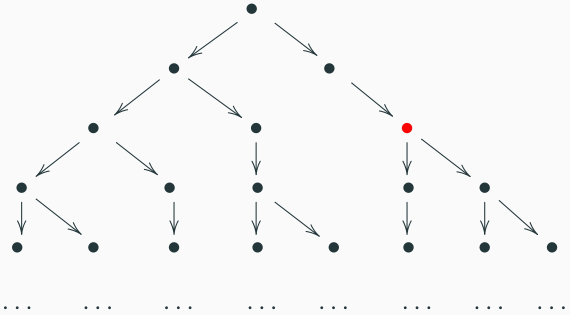

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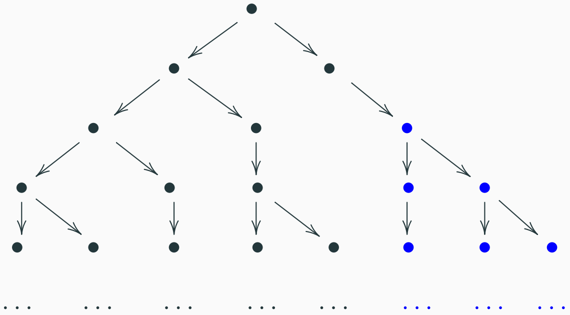

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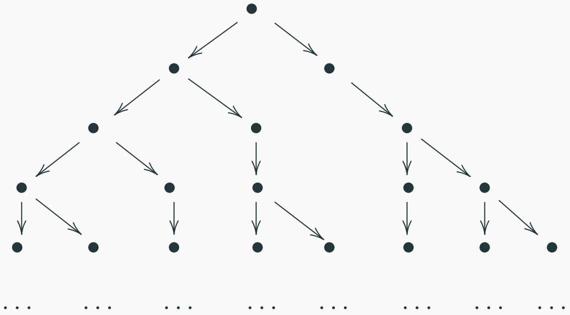

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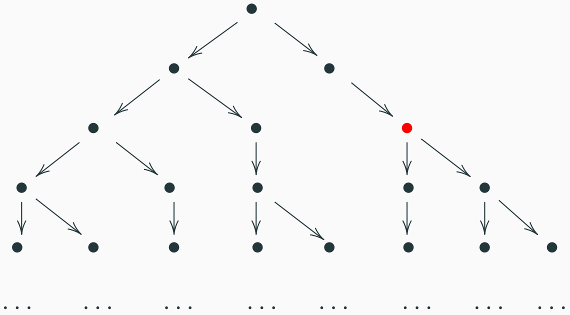

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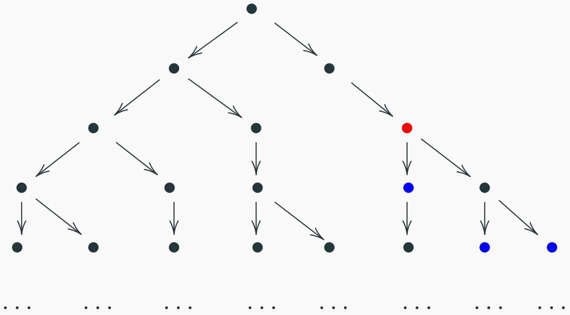

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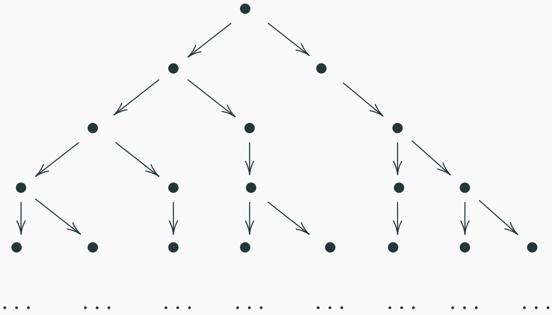

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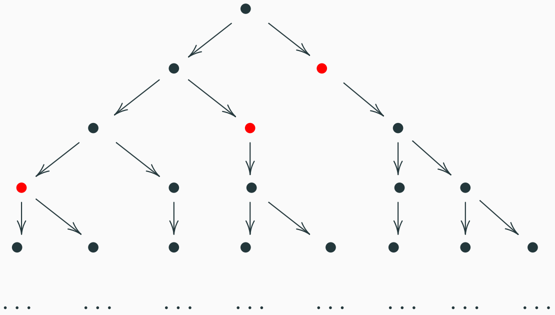

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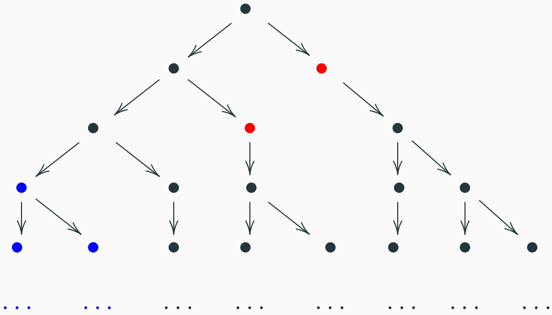

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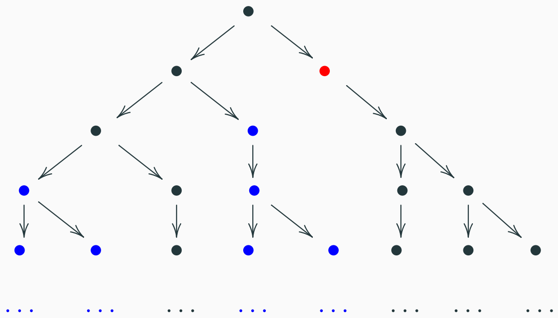

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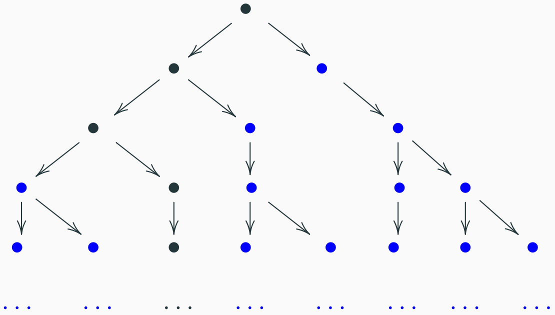

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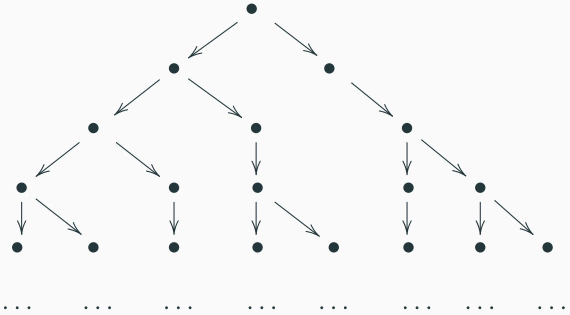

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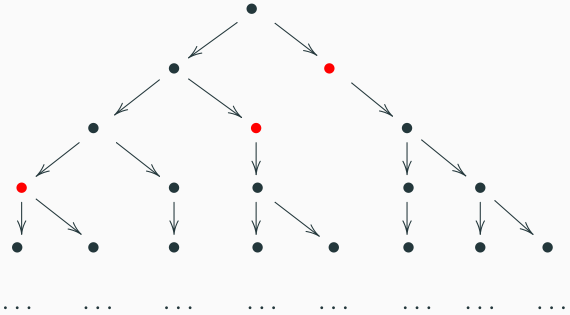

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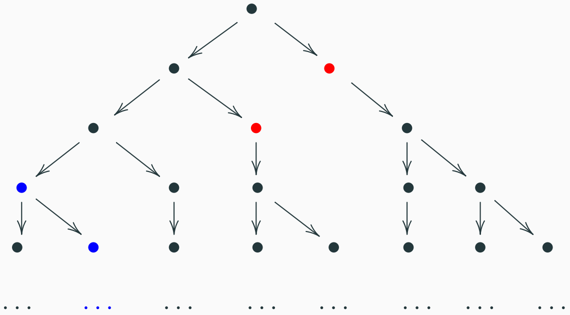

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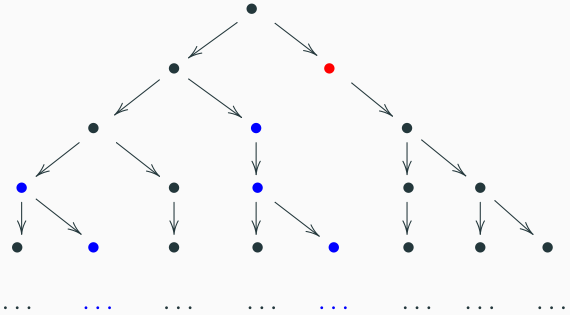

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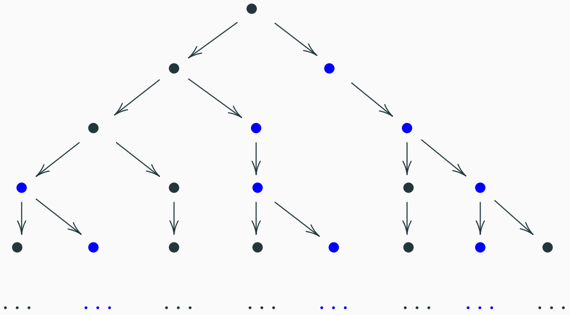

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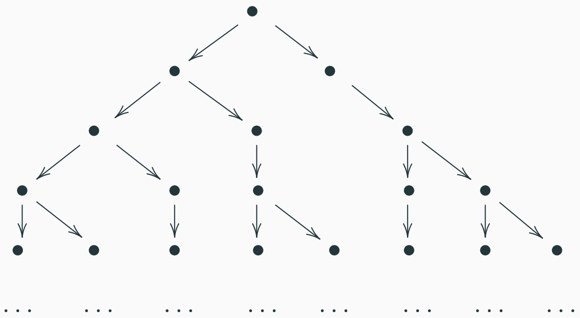

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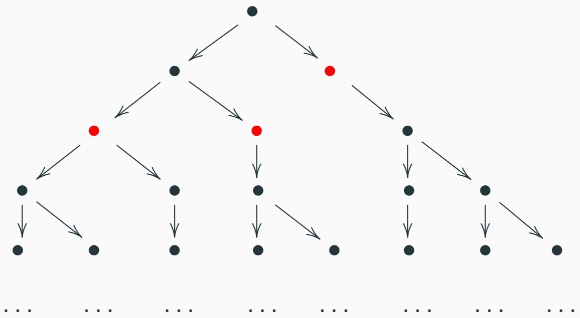

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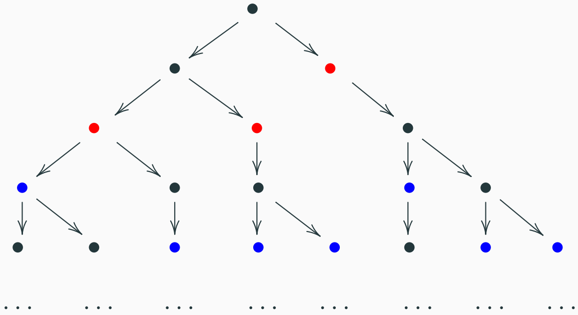

Formal Semantics – The Satisfaction Relation

Given an LTS $\mathcal{M} = (S, \rightarrow, L)$, a state $s \in S$ and a CTL formula φ, we define

"$\mathcal{M}$ in state s satisfies φ" or *"φ holds for $\mathcal{M}$ in state s"*

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by structural recursion on φ:

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$\mathcal{M}, s \models \neg\, \varphi$	iff	$\mathcal{M}, s \not\models \varphi$
$\mathcal{M}, s \models \varphi \vee \psi$	iff	$\mathcal{M}, s \models \varphi$ or $\mathcal{M}, s \models \psi$
$\mathcal{M}, s \models \varphi \wedge \psi$	iff	$\mathcal{M}, s \models \varphi$ and $\mathcal{M}, s \models \psi$
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$\mathcal{M}, s \models \varphi \ \forall\mathsf{U}\ \psi$	iff	for all paths π such that $s_0 = s$ there exists i such that $\mathcal{M}, s_i \models \psi$ and for all $j < i$, $\mathcal{M}, s_j \models \varphi$
$\mathcal{M}, s \models \varphi \ \exists\mathsf{U}\ \psi$	iff	there exists a path π such that $s_0 = s$ and there exists i such that $\mathcal{M}, s_i \models \psi$ and for all $j < i$, $\mathcal{M}, s_j \models \varphi$

Formula Equivalence in CTL

Two CTL formulas φ and ψ are equivalent, denoted $\varphi \equiv \psi$, if they are satisfied by (i.e., hold for) exactly the same LTSs in the same states

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Given any LTS $\mathcal{M} = (S, \rightarrow, L)$ and any $s \in S$, we have that

$$\mathcal{M}, s \models \varphi \text{ iff } \mathcal{M}, s \models \psi.$$

Some CTL Formula Equivalences

Just like for LTL, $\top$ (read "True") abbreviates $a \rightarrow a$ for some atom a.

$$
\begin{aligned}
\exists\Box\forall\Box\varphi &\equiv \forall\Box\forall\Box\varphi \equiv \forall\Box\varphi \\
\neg\exists\bigcirc\varphi &\equiv \forall\bigcirc\neg\varphi \\
\neg\exists\Diamond\varphi &\equiv \forall\Box\neg\varphi \\
\neg\exists\Box\varphi &\equiv \forall\Diamond\neg\varphi \\
\forall\Diamond\varphi &\equiv \top\ \forall\mathsf{U}\ \varphi \\
\exists\Diamond\varphi &\equiv \top\ \exists\mathsf{U}\ \varphi \\
\varphi\ \forall\mathsf{U}\ \psi &\equiv \neg((\neg\psi\ \exists\mathsf{U}\ (\neg\varphi \wedge \neg\psi)) \vee \exists\Box\neg\psi)
\end{aligned}
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Homework Exercise. Apply the CTL semantics to prove the above equivalences.

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Applying (1) to 0, π' and j, we obtain $\mathcal{M}, s'_j \models \varphi$, as desired.

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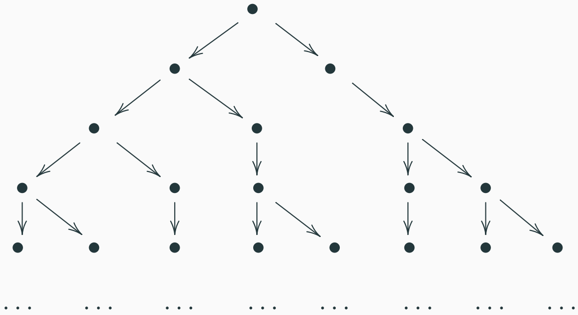

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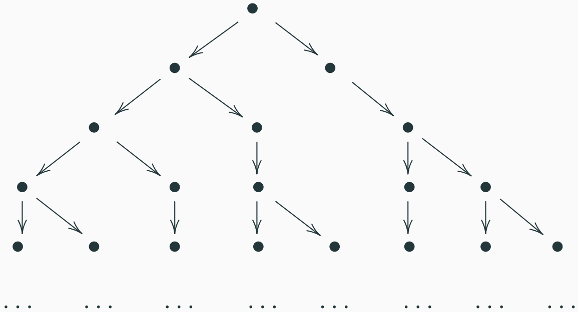

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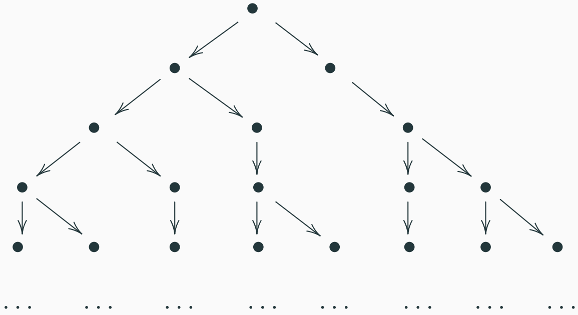

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This concludes our proof of (2) implies (1),
and also our entire proof of $\exists\Box\forall\Box\varphi \equiv \forall\Box\varphi$.

CTL Versus LTL

Differences between LTL and CTL

LTL is limited to questions of the form:

- For all paths, does the LTL formula φ hold?
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Neither CTL can express the first, nor LTL can express the second property.

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Consider an LTS describing two parallel processes, where, for $i \in \{1, 2\}$:

n_i denotes "process i **n**ot in critical section"; r_i denotes "process i **r**equesting to enter critical section"; c_i denotes "process i in **c**ritical section"

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Let φ be a CTL formula, and let ψ be an LTL formula. φ and ψ are said to be equivalent if they are satisfied by (i.e., hold for) exactly the same LTSs in the same states: Given any LTS $\mathcal{M} = (S, \rightarrow, L)$ and any $s \in S$, we have that

$$\mathcal{M}, s \models \varphi \text{ (in CTL) iff } \mathcal{M}, s \models \psi \text{ (in LTL)}.$$

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Let's prove that the CTL formula $\forall\Box\,(\neg(c_1 \wedge c_2))$ is equivalent to (i.e., is satisfied by the same LTSs in the same states as) the LTL formula $\Box\,(\neg(c_1 \wedge c_2))$.

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Remember: π^i is the i'th suffix of π.

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for all $\pi = s_0 s_1 s_2 \ldots \in Paths_{s_0}(\mathcal{M})$, $\pi \models_L \Box\,(r_1 \rightarrow \Diamond c_1)$ in LTL
  iff (by the LTL semantics of $\Box$, on sequences)
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We know:

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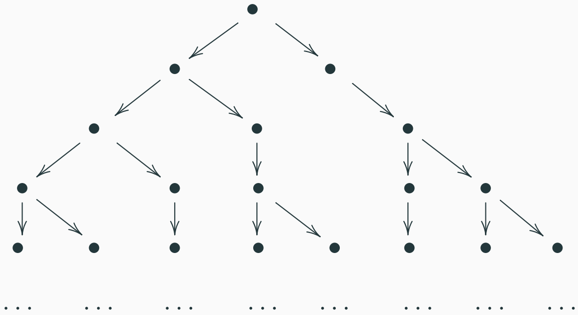

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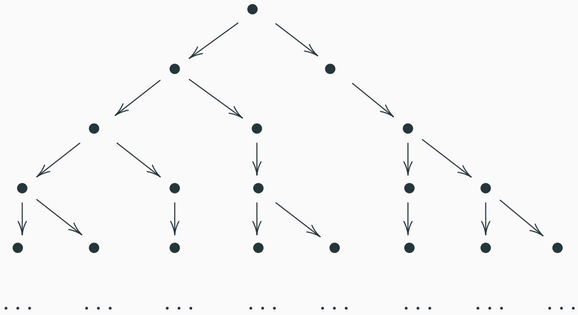
By (2) applied to π'' and i, we obtain j such that $c_1 \in L(s''_{i+j})$, i.e., $c_1 \in L(s'_j)$.

We know:

(2) for all $\pi = s_0 s_1 s_2 \ldots \in \mathit{Paths}_{s_0}(\mathcal{M})$, for all i,
 if $r_1 \in L(s_i)$ then
 there exists j such that $c_1 \in L(s_{i+j})$



We show:

(1) for all $\pi = s_0 s_1 s_2 \ldots \in \mathit{Paths}_{s_0}(\mathcal{M})$, for all i,
 if $r_1 \in L(s_i)$ then
 for all $\pi' = s'_0 s'_1 s'_2 \ldots \in \mathit{Paths}_{s_i}(\mathcal{M})$, there exists j such that $c_1 \in L(s'_j)$

CTL Model Checking (Very Briefly)

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Main lemma: Characterization of the CTL connectives' semantics by means of least and greatest fixpoints of suitable operators on $\mathcal{P}(S)$.

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Not covered in these lectures – see Section 6.4 of Baier & Katoen's "Principles of Model Checking" (MIT Press 2008). Simpler than LTL model checking!

Ending

Summary of the Discussed Concepts

- CTL = Computation Tree Logic
- Syntax = formulas built from
  - atoms
  - propositional connectives
  - CTL connectives, each consisting of a path quantifier and a temporal operator
- Semantics = the satisfaction relation defined on LTSs
- Formula equivalence
- CTL versus LTL
- Brief sketch of CTL model checking

Further Reading

Section 6 of Baier & Katoen's "Principles of Model Checking" (MIT Press 2008)

- Distinguishes state formulas from path formulas
- Writes $\forall(\varphi \mathsf{U} \psi)$ instead of $\varphi \, \forall\mathsf{U} \, \psi$, and $\exists(\varphi \mathsf{U} \psi)$ instead of $\varphi \, \exists\mathsf{U} \, \psi$

Section 3.4 of Huth & Ryan's "Logic in Computer Science: Modelling and Reasoning about Systems" (Cambridge University Press 2004)

Uses different notations for the CTL connectives:

- X instead of $\bigcirc$, F instead of $\Diamond$, G instead of $\Box$ (just like it does for LTL)
- A instead of $\forall$, E instead of $\exists$
- Hence writes AF instead of $\forall\Diamond$, EG instead of $\exists\Box$, etc.
- Also, $\mathsf{A}(\varphi \mathsf{U} \psi)$ instead of $\varphi \, \forall\mathsf{U} \, \psi$, and $\mathsf{E}(\varphi \mathsf{U} \psi)$ instead of $\varphi \, \exists\mathsf{U} \, \psi$